# 16th ICCRTS

# "Collective C² in Multinational Civil-Military Operations"

## A Topological Model for C² Organizations

Topic 2 – Approaches and Organizations

**André Luiz Pimentel Uruguay, Lt.Col., MSc [STUDENT:4th YR PhD Candidate]**

Instituto de Estudos Avançados

Rodovia dos Tamoios, Km 5,5

São José dos Campos, SP 12228-001 - Brazil

auruguay@ieav.cta.br

**Carlos Henrique Costa Ribeiro, PhD**

Instituto Tecnológico de Aeronáutica

Pça Marechal Eduardo Gomes, 50

São José dos Campos, SP 12228-900 - Brazil

carlos@ita.br

*Point of Contact: André Luiz Pimentel Uruguay - Lieutenant Colonel

+55-12-3947-5345 / +55-12-9121-5695 (mobile)

**auruguay@ieav.cta.br**

| 1. REPORT DATE **JUN 2011** | 2. REPORT TYPE | 3. DATES COVERED **00-00-2011 to 00-00-2011** |
|---|---|---|
| 4. TITLE AND SUBTITLE **A Topological Model for C2 Organizations** | | 5a. CONTRACT NUMBER |
| | | 5b. GRANT NUMBER |
| | | 5c. PROGRAM ELEMENT NUMBER |
| 6. AUTHOR(S) | | 5d. PROJECT NUMBER |
| | | 5e. TASK NUMBER |
| | | 5f. WORK UNIT NUMBER |
| 7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) **Instituto de Estudos Avan?os,Rodovia dos Tamoios, Km 5,5,S?Jos?os Campos, SP 12228-001 - Brazil,** | | 8. PERFORMING ORGANIZATION REPORT NUMBER |
| 9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) | | 10. SPONSOR/MONITOR'S ACRONYM(S) |
| | | 11. SPONSOR/MONITOR'S REPORT NUMBER(S) |

12. DISTRIBUTION/AVAILABILITY STATEMENT
**Approved for public release; distribution unlimited**

13. SUPPLEMENTARY NOTES
**Presented at the 16th International Command and Control Research and Technology Symposium (ICCRTS 2011), Qu?c City, Qu?c, Canada, June 21-23, 2011. U.S. Government or Federal Rights License**

14. ABSTRACT
**One essential function in Command is to determine roles, responsibilities and relationships, in order to enable, encourage and constrain certain behaviors. In the context of the complex endeavors expected to happen in the near future, this means to establish a collection of organizations that are fit for the missions. On the other hand, in recent years the mathematical field of Topology has been applied to solve several problems of multiagent coordination, consensus problems, concurrent computing and coverage of sensor networks. The present work aims to introduce some topological methods to the set of problems related to C2 organization. A combinatorial topological model of a C2 organization is presented, based on a topological construct called simplicial complex. It?s expected that the presented model will enable the potential application of results from combinatorial and algebraic topology to problems of organizational design and selforganization.**

15. SUBJECT TERMS

| 16. SECURITY CLASSIFICATION OF: | | | 17. LIMITATION OF ABSTRACT | 18. NUMBER OF PAGES | 19a. NAME OF RESPONSIBLE PERSON |
|---|---|---|---|---|---|
| a. REPORT **unclassified** | b. ABSTRACT **unclassified** | c. THIS PAGE **unclassified** | **Same as Report (SAR)** | **53** | |

**A Topological Model for C² Organizations**

**Abstract**

One essential function in Command is to determine roles, responsibilities and relationships, in order to enable, encourage and constrain certain behaviors. In the context of the complex endeavors expected to happen in the near future, this means to establish a collection of organizations that are fit for the missions. On the other hand, in recent years the mathematical field of Topology has been applied to solve several problems of multi-agent coordination, consensus problems, concurrent computing and coverage of sensor networks. The present work aims to introduce some topological methods to the set of problems related to C2 organization. A combinatorial topological model of a C2 organization is presented, based on a topological construct called simplicial complex. It's expected that the presented model will enable the potential application of results from combinatorial and algebraic topology to problems of organizational design and self-organization.

## 1. Introduction

The goal of development of a community with a shared conceptual framework, sense of immediacy and purpose, and a coherent C2 research program was stated past in the seventies. In a keynote talk presented at the 1987 Symposium of C2 Research, Levis and Athans claimed that, although the attainment of a comprehensive theory of C2 may be a dream, the quest for it is very real. After several meetings and workshops, the evolution of C2 seemed, at that time, hampered by the so called *twin curses*: dimensionality and complexity (Levis & Athans, 1987).

In 2006 Lenahan & Charles proposed a set of 12 grand challenges and associated questions facing the Command and Control community. *Impact* was one of the many criteria chosen for the selection of the most relevant issues, and its definition involved a major difference in terms of better or more agile C2 organizational structures, increased C2 organizational capacity, and increased process influence in terms of "locking out" or dominating adversarial process options through the use of effects projection (Lenahan & Charles, 2006).

We believe that the progress in solving those organization-related issues, by approaching both dimensionality and complexity, is paramount for the true evolution of Command and Control. And this shall be accomplished without recurring to historical metaphors and paradigms (Alberts, 2003).

The dimensionality component of the twin curses derives from the huge state space of the physical, informational, cognitive and social domains of C2 systems. Complexity derives from the interrelationships between these key domains. A theoretical approach to deal with the combinatorial explosion should be the next step in C2 evolution.

This approach should be able to synthesize the combinatorial structures and relationships of the entities performing Command and Control functions. Another important feature would be to rely only on metrics which we can take as of confirmed validity or relevance. This would assure that empirical studies would be conducted only after theoretical results get mature.

The thesis of this article is that a theory of C2 shall be based on representing these interrelationships in the simplest way possible, yet enabling the explanation of complex phenomena. We also claim that the use of the mathematical field of Topology can help not only to solve certain classes of problems involving C2 organizations, but also to enhance the general understanding of Command and Control in a more formal way.

## 2. The Case for a Topological Approach

Many aspects of C2 are hard to quantify. Topology can be referred as a kind of qualitative math, not the math made of points as usual. Instead, it involves sets joined in structures called topological spaces. In general, Topology deals with combinatorial structures and relationships between objects. Topological spaces can show the relationships between sets composing or describing complex systems. And these spaces are not even required to be metric (Euclidean), i.e., they don't need to have a definition of 'distance' between their elements.

Topological thinking is not new in the military domain, since the geographic space is highly influential on tactics, and typical maps of military situation partition the terrain depending on jurisdiction: which organizational unit is responsible for some region. Another example from the information domain is the use of the word 'topology' to describe the structure of network communications infrastructure.

Also, we can use sets to describe behavior, expressed by activities composing processes. And these sets can also be combined with geographic regions and communication network nodes. These sets define a space which, in the military domain, is called *battlespace*,.

Battlespace, according to the DoD definition, is 'the environment, factors, and conditions that must be understood to successfully apply combat power, protect the force, or complete the mission'. This includes the air, land, sea, space, and the included enemy and friendly forces; facilities; weather; terrain; the electromagnetic spectrum; and the information environment within the operational areas and areas of interest.'

These are many distinct aspects that, by the definition, are interrelated and influential to mission success. 'To shape' can mean to conform (or give form) to all these subjects to a configuration more fit to the mission at hand, which is a paramount function of Command and Control.

Another reference to the implicit topological thinking in C2 was given by Alberts and Hayes (2006):

'The most interesting and challenging [C2] endeavors are those that involve a *collection* of military and civilian sovereign entities with *overlapping* interests that can best be met by sharing information and collaboration that *cuts across the boundaries* of the individual entities.'

Intuitions aside, in the seminal article 'Mathematics of Command and Control Analysis', Dockery (1984) expressed his findings about the limitations actually present in C2 analysis: (i) there are no proper investigation tools, (ii) there is no theory and (iii) the treatment of structure has been neglected.

Also, not only the treatment of structure was lacking. The following excerpt gets right to the point:

> 'We have therefore searched for a theory. More fundamentally, we have searched for a starting point for a theory. In the end we focused on structural aspects of C2. In fact structure is but one of three aspects of the problem which we have identified. The other two are data/information and transactions. The complete characterization is therefore transactions within a structure involving the flow of data/information through that structure.'

On another front, in the knowledge domain of multiagent systems research, an organization is viewed as a set of constraints accepted by a group of agents in order to facilitate their goals' achievements, i.e., an agent must limit its autonomy in order to comply with the structure and function of an organization.

For instance, the organizational model for multiagent systems MOISE+ is based on three different specifications: structural, functional and deontic. Fig.1a shows how the structural and functional dimensions, jointly with the environment, can explain or constrain the organizational behavior in trying to accomplish its social purposes.

Another example is the model depicted in Fig.1b, where the basic premise is that rational organizations will always try to match sets containing the current and desired states of the organization, as well as the scope of control of its members.



(a) Behavior Space - Example 1 (HÜBNER; SICHMAN; BOISSIER, 2004)  (b) Behavior Space - Example 2 (DIGNUM; DIGNUM, 2007)
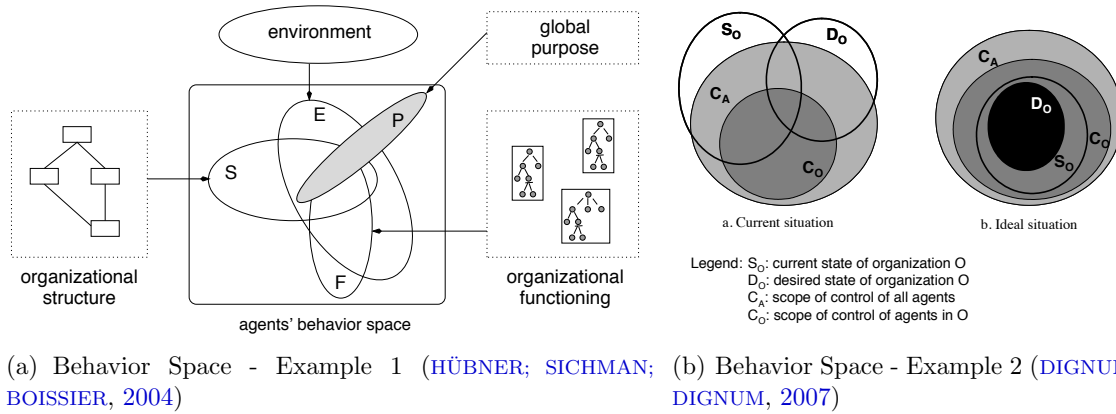
FIGURE 1 – Two examples of intuitive topological thinking about the behavior of organizations.

In summary, all these models commonly illustrate constraints of behavior using Venn diagrams, showing the interplay of sets representing the external environment, the structure and functions of the organization, and the capabilities of its members, as these sets somehow define the boundaries of organizational performance and the eventual need to reorganize.

Notwithstanding the relevance and recurrence of this intuitive representations, to our knowledge no work was found trying to formalize the interrelationship of these sets of elements as determinants of organizational behavior. Also, the effects of the organizational structure on the performance are still not understood.

This common use of sets to express behavior points to the question of an organization being represented *topologically*. Topology is the area of Mathematics responsible for the study of properties of spaces that are preserved under continuous deformations, having been emerged through the development of concepts from geometry and set theory. Roughly speaking, Topology aims to provide qualitative thinking about sets and their relationships.

Thus, topological notions like compactness, connectedness and denseness are as important nowadays as sets and numbers were for the mathematicians of the past.

## 3. Basic Definitions

As already mentioned, Topology involves sets joined in structures called topological spaces. In general, Topology deals with combinatorial structures and relationships between objects. Algebraic Topology, in special, is the study of spaces and maps using algebraic methods. The main idea for Algebraic Topology is to map problems that would be rather difficult to solve using Topology, to an algebraic form, where abstract algebra methods can more easily be applied. Thus, some problems can profit from moving from a representation based on spaces and maps to another, composed of classic algebraic constructs.

Results from algebraic topology suggest that a lack of relative or global localization in a network is not an obstruction to determining global network features. For example, recent results indicate that homology theory provides a powerful yet computable set of criteria for coverage in sensor networks (SILVA; GHRIST, 2006). It is possible that many of the tools developed by topologists over the past century for passing from local combinatorial data to global topological data may provide valuable insights in extracting a global picture from nodes with local communication links.

For the goals of this work what is required is to present two basic concepts necessary to understand the results of previous work, and how these results support the feasibility of this approach.

**Definition**: A *Topological Space* is a set X together with a collection O of subsets of X, called open sets, such that:

- The union of any collection of sets in O is in O;

- The intersection of any finite collection of sets in O is in O; and
- Both $\varnothing$ and X are in O.

The collection O is called a topology on X.

One important combinatorial construct for the techniques that will follow is the simplicial complex (COOMBS; JARRAH; LAUBENBACHER, 2001):

**Definition**: An *Abstract Simplicial Complex* $\Delta$ on a finite set $V$, whose elements are called vertices, is a non-empty collection of subsets of $V$ that is closed under taking subsets, i.e., if a subset of $V$ belongs to $\Delta$, then all its subsets also belong to $\Delta$. The elements of $\Delta$ are called simplices or faces. The dimension of a simplex $\sigma = \{v0,...,vn\} \in \Delta$ with $n$ elements is $n$, being defined a *n*-simplex. The dimension of $\Delta$ is the maximum of the dimensions of its simplices.

Another alternative, shorter definition is (JONSSON, 2008):

**Definition**: An *Abstract Simplicial Complex* $\Delta$ on a finite set $X$ is a family of subsets of $X$ closed under deletion of elements. We refer to the singleton sets $x$ in $\Delta$ as *0*−simplices or vertices. It is not required that $x \in \Delta$ for all $x \in X$.

Fig.2 shows examples of simplices of dimension 0 to 3.
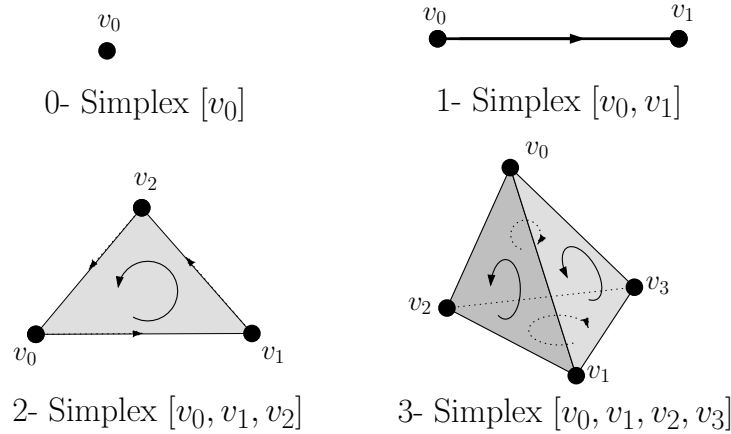


FIGURE 2 – Oriented Simplices of Dimension 0 to 3 (Ghrist & Muhammad,2005).

Example:  Let $V$ = *{A, B, C, D, E, F }*, and let $\Delta$ be an abstract simplicial complex composed of the subsets *{A, B, C}*, *{B, C, D, E}*, *{C, F}*, *{E, F}* and their subsets. This abstract complex can have a geometric representation, depicted in fig.3.
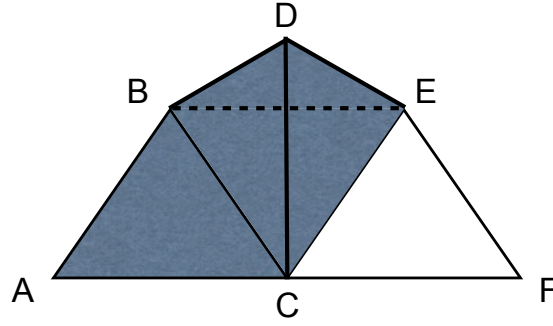
FIGURE 3 – An example geometric representation of a 3-dimensional abstract simplicial complex.

Its dimension is 3, as there is a 3-dimensional simplex ({B, C, D, E}), in addition to one 2-dimensional and two 1-dimensional simplices. It's worth to notice that, although the example contains the sets {C,E}, {C,F} and {E,F}, the set {C,E,F} is not part of the complex, leaving a 'hole' in the geometric representation. On the other hand, the simplex {A, B, C} has its interior 'filled', i.e., this simplex captures not only a set of three 1-dimensional edges, but also a special, higher dimensional new kind of 'edge', relating vertices *A, B* and *C*.

Another form of representation of a simplicial complex is using matrices. Let us take an incidence matrix where the columns are the vertices (the 0-dimension simplices) of the complex, and the rows are the simplices, being enough to represent only the maximal simplices with respect to inclusion. In the previous example, this matrix would be

$$
\begin{matrix} A & B & C & D & E & F \end{matrix}
$$

$$
\begin{pmatrix}
0 & 0 & 1 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 1 & 1 \\
1 & 1 & 1 & 0 & 0 & 0 \\
0 & 1 & 1 & 1 & 1 & 0
\end{pmatrix}
$$

Simplicial complexes are purely combinatorial objects, with geometric representations, which can therefore be viewed as topological spaces. They are combinatorial versions of topological spaces and their relevance resides in the fact that they can be analyzed with combinatorial, topological and algebraic methods.

## 4. Conceptual Model

The purpose of these conceptual model is to summarize a simple model of organizations to allow further construction of some combinatorial topological objects. In

general, organizational models are composed of structural and functional descriptions, as to mimic the conventional way of analyzing a system, either looking at its form or its function. Also, to the model be complete, the relationships between structure and function must be specified.

The structural part defines roles as the elementary components of the organization, and also the relationships between roles and between groups of roles. There is discussion on the possible types of relationships as authority, communication (the right to communicate with) and acquaintance (Hubner; Sichman; Boissier, 2007), as also power, coordination and control (Grossi et al., 2006).

The functional part specifies the functions an organization shall perform in order to accomplish its mission. As with the structure, there are many ways to represent this description:global goals and plans (Hubner;Sichman;Boissier,2004), or IDEF0 (Usa, 1998) functions (Uruguay; Hirata, 2006).

In order to completely express contexts for reorganization, most computational models are composed of agents, roles, tasks, goals, resources and capabilities. In the present work these elements were summarized to:

• *Roles*: express the constraints an agent has to accept on its own behavior, in order to be part of the organization;
• *Relationships*: how two roles interact one with each other. The nature of the relationship (command, coordination, acquaintance) for now is not relevant;
• *Tasks*: represent the activities an agent commit to perform. These can generate some effect on the environment or just be internal to the agent.

The relevance of other concepts commonly found in organizational models is not under question here. In general, concepts like costs, resources and capabilities are important to better define the merit of an organization, understood as the congruence to its mission, and to drive agents in eventual efforts to reorganize themselves. But as the main purpose of the article is to represent organizational behavior topologically, and not to propose a new reorganization strategy, we consider these concepts to suffice so far.

Two points are worthy of note here: many models express explicitly the idea of a Group. Although the organization can specify groups, many reasons can let an agent to join (or to leave) an organization. It's not guaranteed that the membership of a group will always be the main source of motivation for agents to enter an organization.

Another point is about Tasks and Goals. Both Tasks and Goals can be decomposed, at least for analytical purposes. In hierarchical structures, part of this decomposition can result from the deliberation of superior roles, during the assignment of tasks to their subordinates. However, as an agent committed to a role preserves its autonomy (at least

partially), it can make further task decompositions in subgoals as part of its planning, committing itself to these new goals. As both effects are part of the behavior of the organization as a whole, and to the purpose of representing behavior topologically, in this work we simplify using the term 'task' to represent both concepts.

To clarify using another perspective, distinction must be made between functional and procedural (or process) models. Functions are essentially the desired result of a task (or set of tasks). Processes describe explicit sequences of tasks, with some models even including timing and scheduling considerations. The sequential dependencies included in the adopted representation for the functional model are not to be taken as a process, but, instead, as a constraint on the space of candidate processes to perform a certain functional model, being up to the agent to plan and to decide which process it will try.

With those primary concepts at hand, we can state our organizational model as follows. An organization is specified as

$$O = (O_{struct}, O_{func}, O_{assign})$$
$$O_{struct} = (R, Rel)$$
$$O_{func} = (T, P(T))$$
$$O_{assign} = \{(r_i, \{t_i)\} | r_i \in R, t_i \in T\}$$

$O_{struct}$ is the structure of the organization, comprising the set $R$ of its actual members, and how they are related to each other, expressed by the poset $Rel$. $O_{func}$ states the functional model: the set of tasks $T$ the organization must perform in order to fulfill its mission, and a partially ordered set (poset) $P(T)$, expressing the dependencies between tasks. Finally, $O_{assign}$ completes the description, as the actual assignment of a set of tasks to each role.

The behavior can be captured by the dynamics of the state of the organization. To represent these dynamics, we based our model on some concepts of previous work made by Matson and DeLoach (MATSON; BHATNAGAR, 2006b; MATSON, 2009).

The original definition of organizational state as defined by Matson and DeLoach is

$$O_{state} = (A, POS, CAP, ASN)$$

where A is a set of agents, POS is a function describing the capabilities of each agent, CAP is a function stating at which degree an agent is capable of performing a certain role, and ASN is an assignment function relating agents to roles and capabilities.

Focusing on behavior, and for now putting aside the concept of capabilities, the state of the organization is summarized by the roles it actually contains, and the tasks actually being performed by each role:

$$O_{behaviorstate} = (r_{i,k}, \{t_{i,k}\})$$

where $(r_{i,k}, \{t_{i,k}\})$ expresses that agent $r_{i,k}$ is performing the set of tasks $\{t_{i,k}\}$ at instant k. As tasks are accomplished (or not), the state of behavior transitions to new tasks being started by the agents:

$$O_{behaviorstate,k} \longrightarrow O_{behaviorstate,k+1}$$

## 5. Topological Model

The proposed topological model to represent the behavior of an organization is derived in a two-step process. To better describe it, we use a notional model for a C² organization in charge of Air Defense (AD) mission.

The organization is composed of four basic roles: *Central Command Post* (**CC**), *Local Command Post* (**LC**), *Fighter Aircraft* (**F**) and *Anti-Aircraft Artillery* (**AAA**). Also, there are five basic tasks involved:

- *Detect and Identify* (**DI**): to detect unauthorized aircraft entering airspace under control of the organization;
- *Order to Engage* (**OE**): to order an asset (Fighter or Anti-Aircraft Artillery) to attack the intruder aircraft;
- *Intercept* (**IN**): to take-off and proceed to intercept intruder;
- *Engage Artillery* (**EA**): to attack the intruder using AAA; and
- *Report Results* (**RR**): to report the results of engagements, be them interceptions or artillery attacks.

The number of tasks was kept deliberately short in order to keep the task dependency graph with paths of no more than 3 hops long and, thus, to allow the reader to visualize the simplicial complexes geometrically, in 3-dimensional figures.

Formally, an initial organizational model could be

$$O_{struct} : (\{CC,LC,F,AAA\},\{CC < LC,LC < F,CC < AAA\})$$
$$O_{func} : (\{DI,OE,IN,EA,RR\},\{DI < OE,OE < IN,OE < EA,IN < RR,EA < RR\})$$
$$O_{assign} : \{(CC, \{DI, OE\}), (LC, \varnothing), (F, \{IN, RR\}), (AAA, \{EA, RR\})\}$$

With this tiny organization we now can proceed in two steps to construct a topological model of its behavior. For this we build on previous work by Coombs, Jarrah and Laubenbacher (COOMBS; JARRAH; LAUBENBACHER, 2001).

The first step is to represent its dynamics. From the functional specification we can derive the partially-ordered set (*poset*) representing the task dependency graph, shown, for this case, in Fig.3.

CC        LC        F        AAA

DI

OE

IN        EA

RR        RR
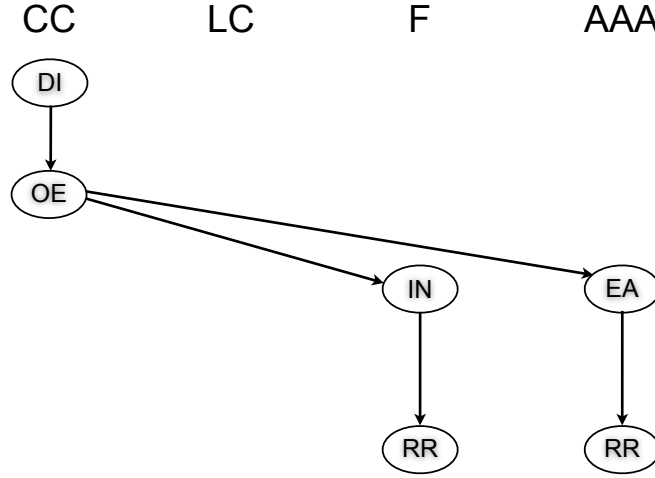
FIGURE 3 – Interaction Poset for the current example. Arrows represent dependencies between tasks, derived from the functional specification of the organization.

This sequence, depicted in Fig.3 defines a partially ordered set P called interaction poset, jointly resultant of the tasks' interdependencies and their assignments to roles. This poset represents the combined effect of functional specification, tasks' assignment and autonomous decisions taken by the agents.

**Definition**: an *Order Complex* $\Delta_o(P)$ is a simplicial complex whose vertex set contains all elements of the interaction poset *P*. A subset of *P* is a simplex of $\Delta_o(P)$ if and only if its elements form a chain in *P*, that is, they can be arranged to form a totally ordered subset of *P*.

The second and final step is to build the order complex. For this example the complex is as follows:

$$\Delta_o(P) = \{\{(CC,DI),(CC,OE),(F,IN),(F,RR)\}, \\ \{(CC, DI), (CC, OE), (AAA, EA), (AAA, RR)\}\}$$

The resultant complex is, then, composed of two 3-dimensional simplices, as shown in Fig.4. Geometrically, the complex is represented by two tetrahedrons, with interior volume and faces filled, i.e., the geometric complex encompasses not only its edges, but also the area of the faces and the interior volume. Also, both 3-simplices share one 1-dimensional face, {(CC, DI), (CC, OE), }.
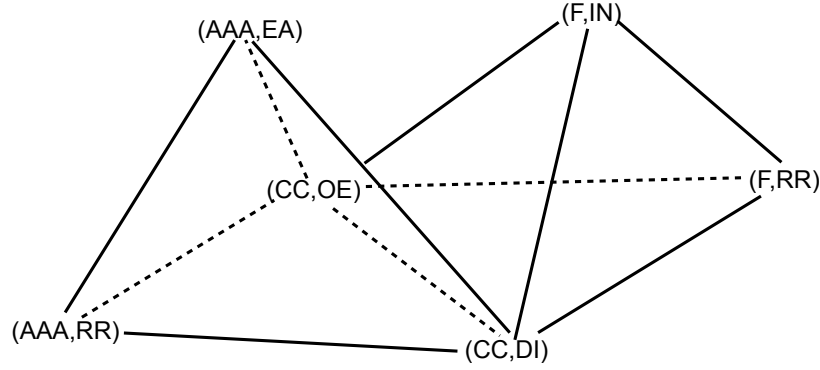
FIGURE 4 – Order Complex for the current example.

At this point it's worth to note that this representation does not mean to express fixed, rigidly defined processes. Instead, the main goal is to demonstrate that any change in the process (let's say, a fighter going rogue and acting fully autonomously), represented by its poset, can be reflected in the topological order complex.

As an example, let's show how a change in the task allocations could impact the topology of the resulting order complex. Let us say that the Central Command Post (CC) delegates to agents performing the role of Local Command Post (LC) the responsibility for ordering the Fighter (F) to intercept intruder aircrafts. Now the new organizational model is

$$O_{struct} : (\{CC,LC,F,AAA\},\{CC < LC,LC < F,CC < AAA\})$$
$$O_{func} : (\{DI,OE,IN,EA,RR\},\{DI < OE,OE < IN,OE < EA,IN < RR,EA < RR\})$$
$$O_{assign} : \{(CC, \{DI, OE\}), (LC, \{OE\}), (F, \{IN, RR\}), (AAA, \{EA, RR\})\}$$

The dynamics is changed, so that the new interaction poset is as shown in Fig.5.
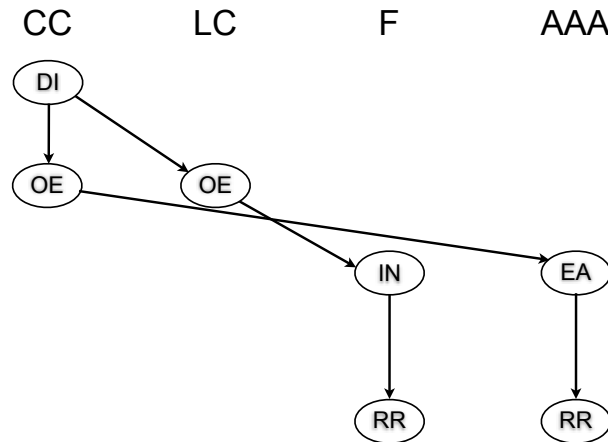


FIGURE 5 – New interaction poset for the current example. Task OE was partially delegated to role LC.

The new order complex now changed to

$$\Delta_o(P) = \{\{(CC,DI),(LC,OE),(F,IN),(F,RR)\},$$
$$\{(CC, DI), (CC, OE), (AAA, EA), (AAA, RR)\}\}$$

The resultant order complex is still composed of two 3-dimensional simplices, as shown in Fig.6, but now both simplices share only a 0-dimensional simplex, the vertex (CC, DI).
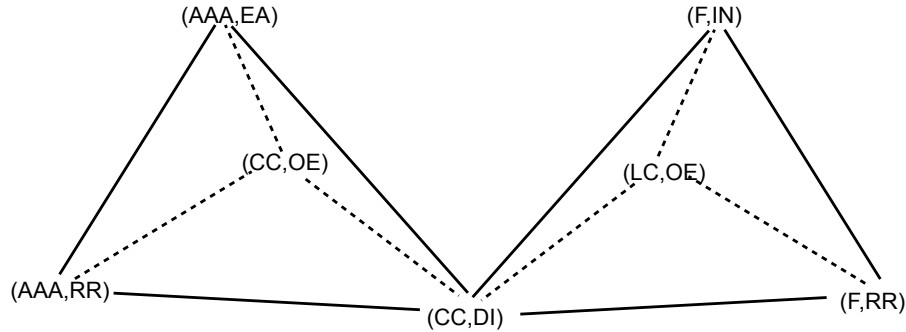
FIGURE 6 – The new order complex after task delegation.

We can proceed even further in delegating to Local Command Post (LC) also the task of commanding agents performing the role Anti-Aircraft Artillery (AAA):

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$$O_{struct} : (\{CC,LC,F,AAA\},\{CC < LC,LC < F,CC < AAA\})$$
$$O_{func} : (\{DI,OE,IN,EA,RR\},\{DI < OE,OE < IN,OE < EA,IN < RR,EA < RR\})$$
$$O_{assign} : \{(CC, \{DI\}), (LC, \{OE\}), (F, \{IN, RR\}), (AAA, \{EA, RR\})\}$$

The new interaction poset is as shown in Fig.7.
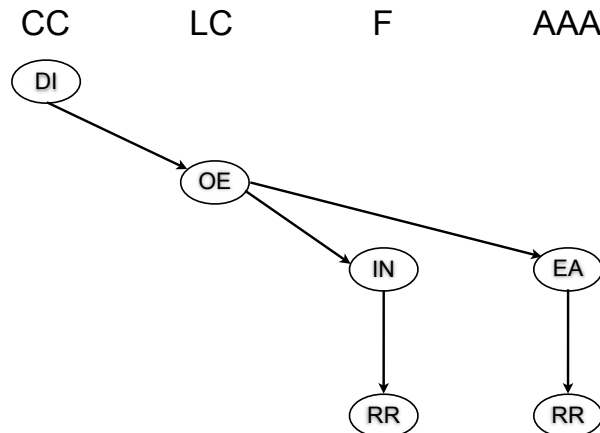
FIGURE 7 – New Interaction Poset for the current example. Now task OE was fully delegated to role LC.

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The new order complex now changed to

$$\Delta_o(P) = \{\{(CC,DI),(LC,OE),(F,IN),(F,RR)\},$$
$$\{(CC, DI), (LC, OE), (AAA, EA), (AAA, RR)\}\}$$

and the resultant order complex is presented in Fig.8. The two 3-dimensional simplices now share a 0-dimensional face, different from the original (edge ({(CC, DI), (LC, OE)})). This fact shows, topologically, that the LC role now is in two different paths of interaction.
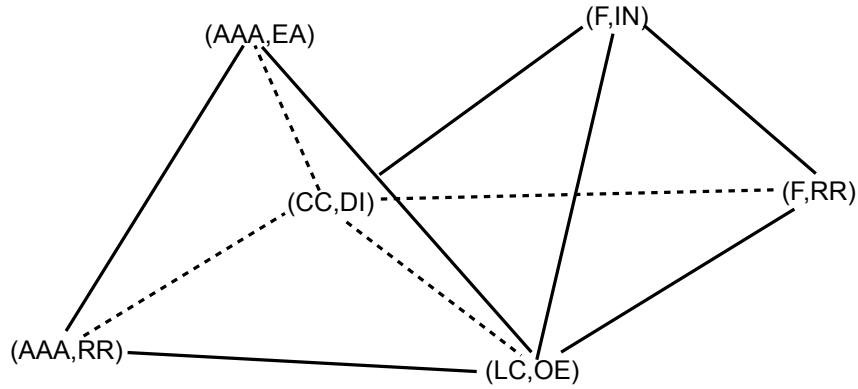


FIGURE 8 – The new order complex after task delegation.

As a last case, let us say that Anti-Aircraft Artillery now has its own resources and authority that it takes to fulfill its mission, i.e., it can detect, identify, decide and act accordingly, and independently from Central and Local Command Posts. This new organization is

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*Ostruct : ({CC,LC,F,AAA},{CC < LC,LC < F,CC < AAA})*
*Ofunc : ({DI,OE,IN,EA,RR},{DI < OE,OE < IN,OE < EA,IN < RR,EA < RR})*
*Oassign : {(CC, {DI}), (LC, {OE}), (F, {IN, RR}), (AAA, {DI, OE, EA, RR})}*

The new interaction poset is as shown in Fig.9. The new order complex now changed to

CC          LC          F          AAA
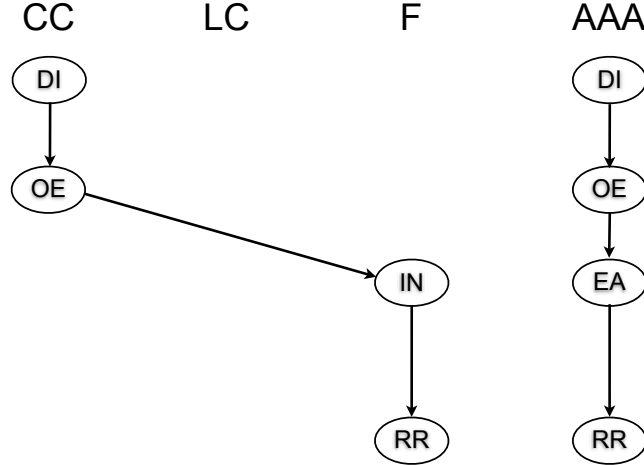
FIGURE 9 – New Interaction Poset for the independent AAA case.

$$\Delta o(P) = \{\{(CC,DI),(LC,OE),(F,IN),(F,RR)\},$$
$$\{(AAA,DI), (AAA,OE), (AAA, EA), (AAA, RR)\}\}$$

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and the resultant order complex is presented in Fig.10. The two 3-dimensional simplices now share a 0-dimensional face, different from the original (edge ({(CC, DI), (LC, OE)}}). This fact shows, topologically, that there are two different paths of interaction.

(F,IN)          (AAA,EA)

(CC,DI)          (AAA,DI)

(CC,OE)          (AAA,OE)

(F,RR)          (AAA,RR)
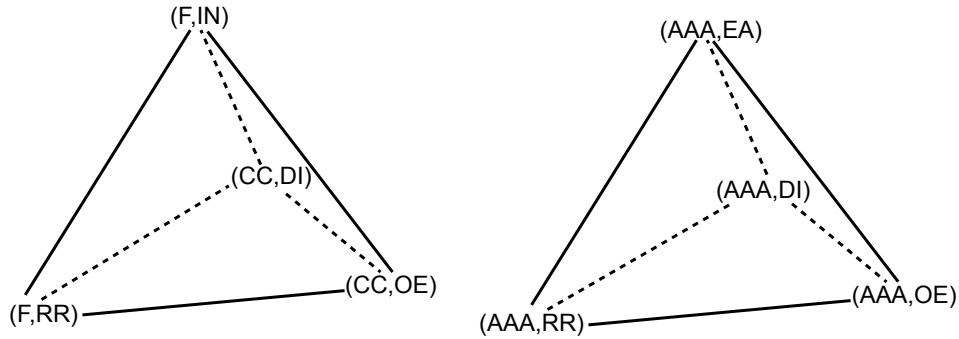
FIGURE 10 – The new order complex for the independent AAA case.

The results obtained so far are aimed at the construction of a topological organizational model encompassing structural and functional specifications. To complete this milestone the present model will be enhanced with other parameters, e.g., timing and information flow, in order to represent more complex organizations.

Monday, November 8, 2010

About the external constraints over a C2 organization, we can imagine several types of environment organizations could be performing in. For this goal one intended approach is to take the geographic space as a notional model, since many application domains involve actors operating in this space. In particular, Command and Control of military

15

forces is one such case, being the physical environment one of the key domains of the network-centric warfare conceptual model (Evidence Based Research, Inc., 2003).

## 6. Conclusions

A process to build a combinatorial topological object from the dynamics of an organization was presented. Although the adopted model of organization is rather simplistic, the applicability of the method for several models is assured, by taking common concepts, like roles and tasks.

The process results in a simplicial complex that captures the functional distribution of work in the organization, by assigning a set of task for each role. Also, the dependency between tasks is expressed, constraining the behavior of the agents committed to perform the roles established by the organization.

It is demonstrated how changes in dependencies between tasks can impact the topology of the resulting complex. By adding or removing dependencies the connectivity of the resulting order complex is changed. Although reorganization processes are out of the scope, it's expected that these changes will condition the way possible strategies to reorganize could be implemented.

Another important aspect is autonomy. For an agent performing a role, the respective set of assigned tasks become its goals. The further activities done by the agent in pursuit of those goals are, for the agent itself, tasks. The topological model doesn't make any distinction between organization tasks and agent tasks and, doing so, is immune to variations in the degree of autonomy. Provided that each agent enacting a role must comply with the structural and normative constraints of the organization, the agent is completely autonomous to deliberate how it will pursuit that function, thus creating its own process.

Also, information requirements can typically be a source of dependencies between tasks. Normally information exchange is done by communication networks, which, by itself, represents another class of topological spaces able to influence organizational behavior.

Finally, no matter which model we adopt to represent organizations, the main tenets of this work are: (i) any model will have to deal with parameters derived from several dimensions, e.g., structure, functions, environment, cognition, capabilities and resources, to name just a few; (ii) the relationships between these parameters can not be captured in a graph-style, one dimension only; and (iii) simplicial complexes are an adequate mathematical construct to capture these higher dimensional relationships.

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# A TOPOLOGICAL MODEL OF C² ORGANIZATIONS

*Lt.Col. André Luiz Pimentel Uruguay, PhD Student*
Instituto de Estudos Avançados - Brazilian Air Force
**auruguay@gmail.com**
*Prof. Carlos Henrique Costa Ribeiro, PhD*
Instituto Tecnológico de Aeronáutica - Brazilian Air Force
**carlos@ita.br**

# Let us recall our first Math classes...

*First, we learn about **sets**....*

V = { }

F

1
2    3

1881 John Venn's original diagram

NAMES OF PEOPLE    ME!    NAMES OF DIAGRAMS

AS REVEALED IN **EARTH** (THE BOOK)
*The Daily Show with Jon Stewart presents*
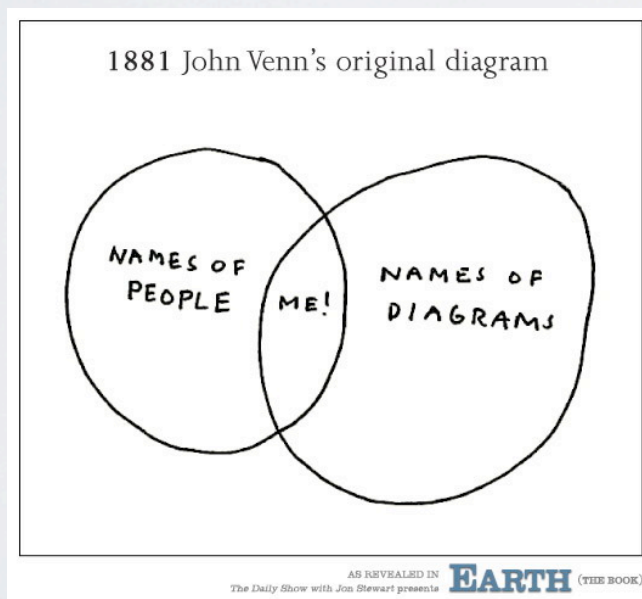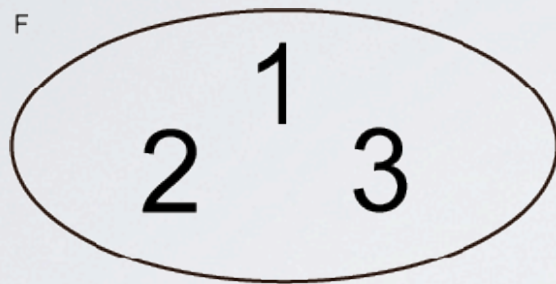
*...to be able to understand **numbers**!!!*

# Let us recall our first Math classes...

## *But once we learn **numbers**...*

Let us recall our first Math classes...

*But once we learn **numbers**...*
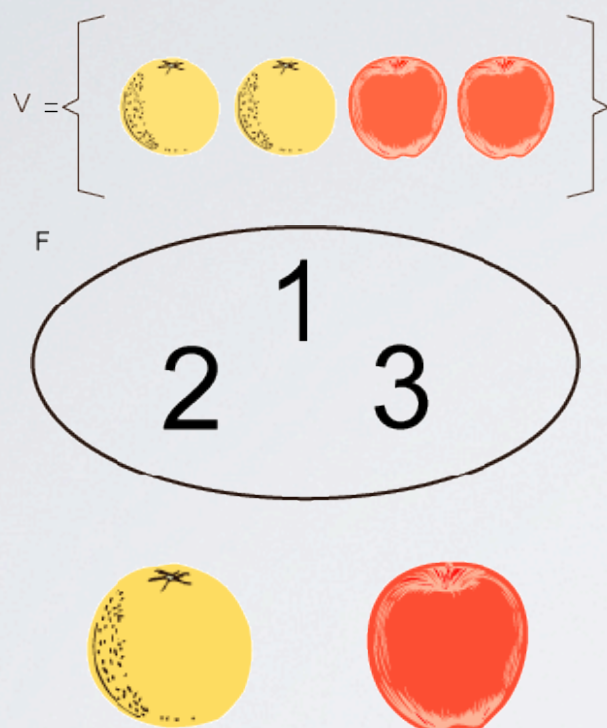


| + | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 2 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| 3 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 4 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
| 5 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
| 6 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
| 7 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
| 8 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 |
| 9 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |

*...we forget **sets** for the rest of our lives!!!*

# OBJECTIVES

- *To motivate the $C^2$ research community for the application of Topology to $C^2$ theory building*

- *To present a topological model of $C^2$ organizations*

# SUMMARY

- Topology in the military

- Topology in $C^2$

- Definitions

- The *Model*

# TOPOLOGY

- From the Greek **τόπος**, "place", and **λόγος**, "study"

- Concerned with properties of objects preserved under continuous deformation

- '*What's the difference between a mug and a donut?*'
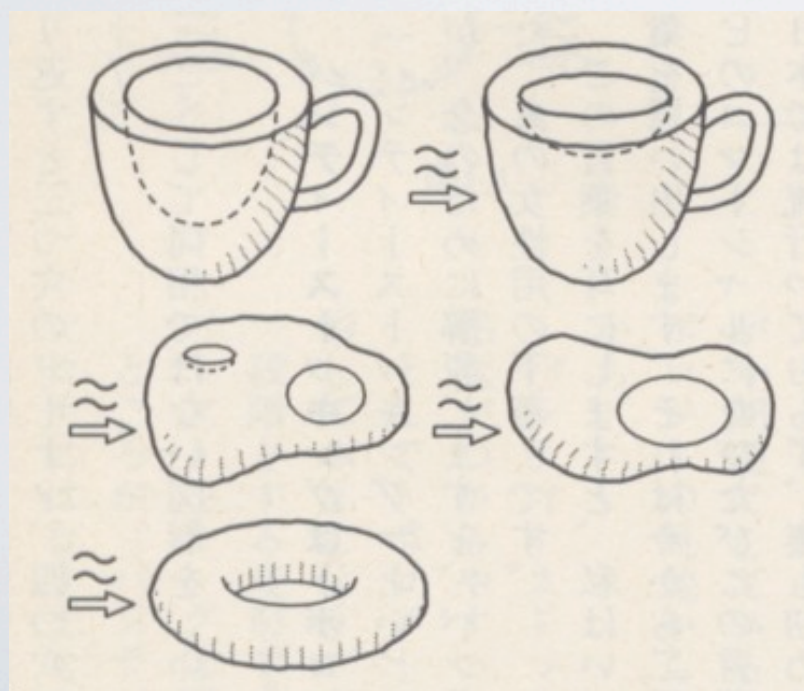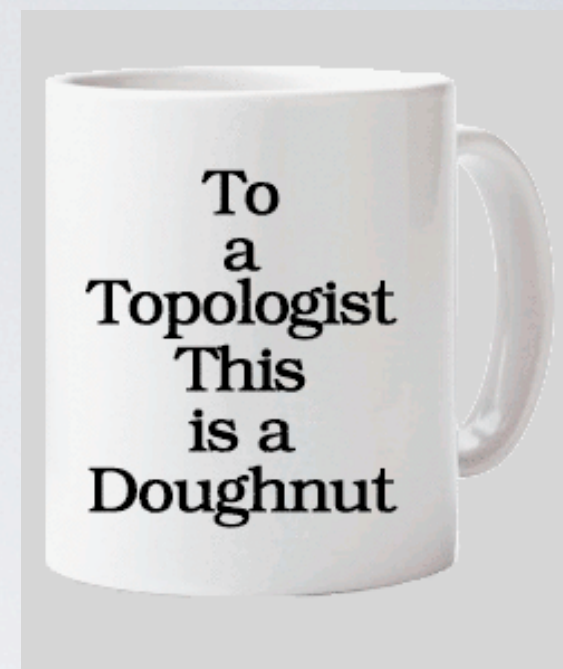
- For the point of interest to <u>Command and Control</u>:

  - Structures of complex objects and their combinatorial relationships

  - Combinatorial Topology (older name for Algebraic Topology)

# TOPOLOGY IN AI

- From research on organizations of multiagent systems:

- *Organizational Congruence* is intuitively expressed as topological constructs

- 2 examples below:



(a) Behavior Space - Example 1 (HÜBNER; SICHMAN; BOISSIER, 2004)

(b) Behavior Space - Example 2 (DIGNUM; DIGNUM, 2007)
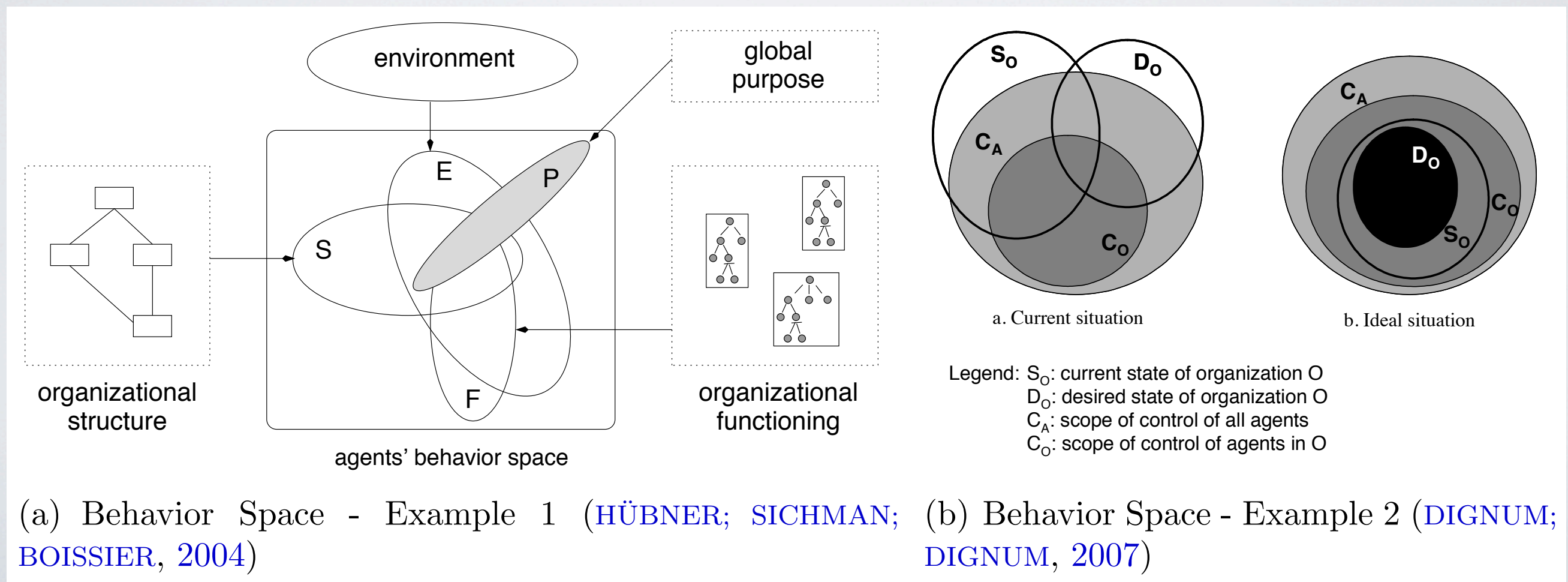
# TOPOLOGY IN THE MILITARY

- (DoD definition) **Battlespace**: 'the environment, factors, and conditions that must be understood to successfully apply combat power, protect the force, or complete the mission'. This includes the air, land, sea, space, and the included enemy and friendly forces; facilities; weather; terrain; the electromagnetic spectrum; and the information environment within the operational areas and areas of interest.'

- *'To shape the battlespace'*: to *conform* or to *give form* to all these elements to a configuration better fit to the mission

# TOPOLOGY IN THE MILITARY

- Geographic space

- Situation maps

- Communication networks
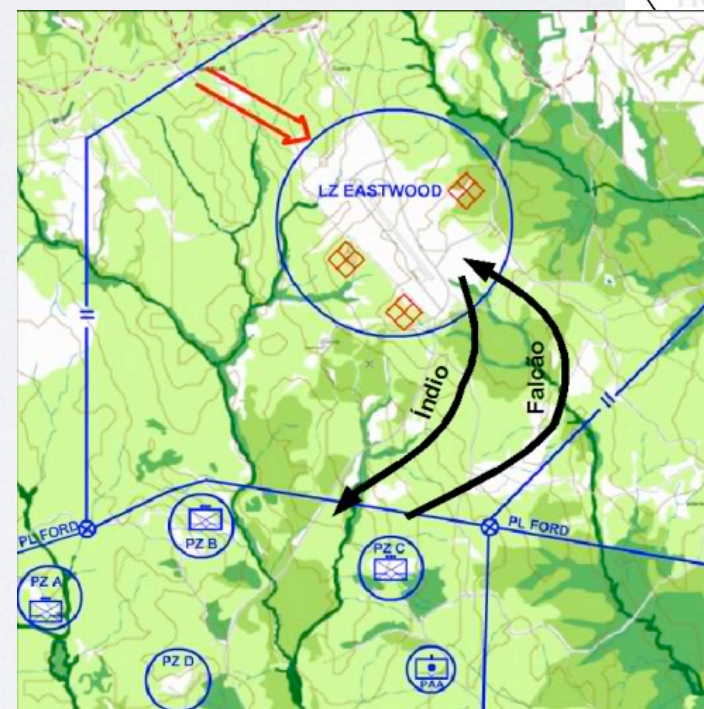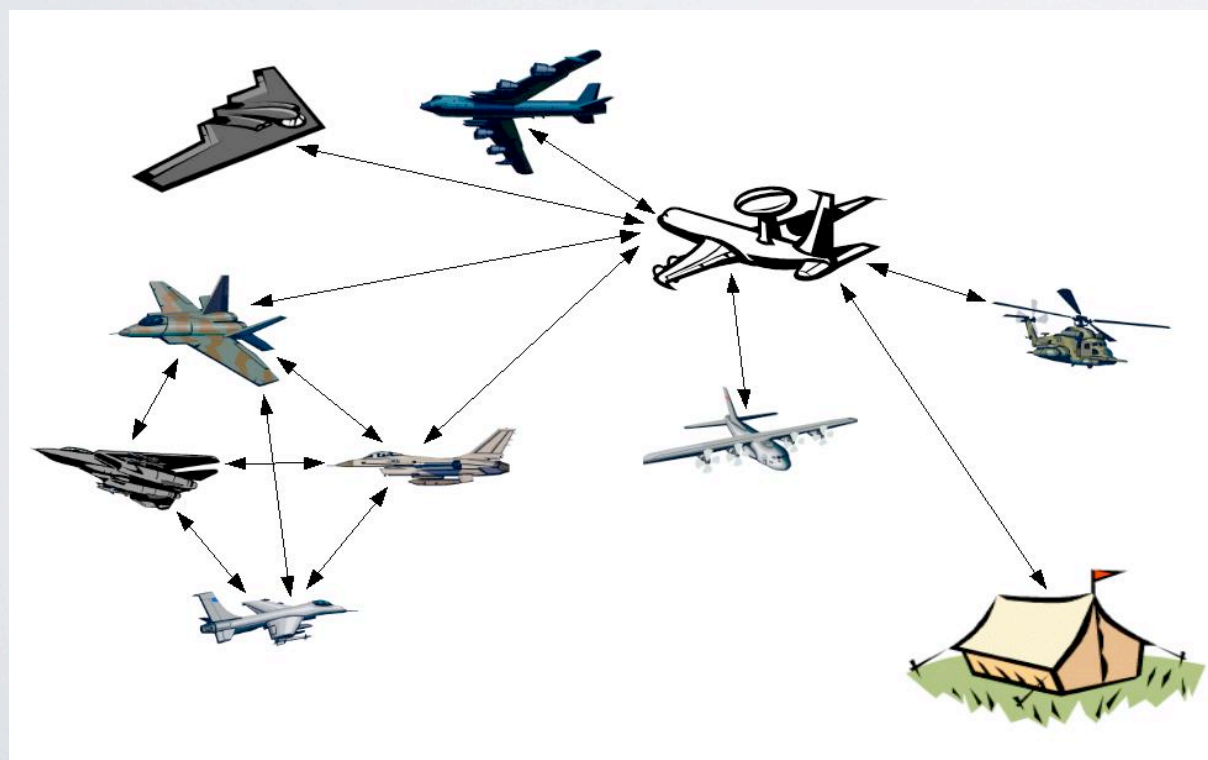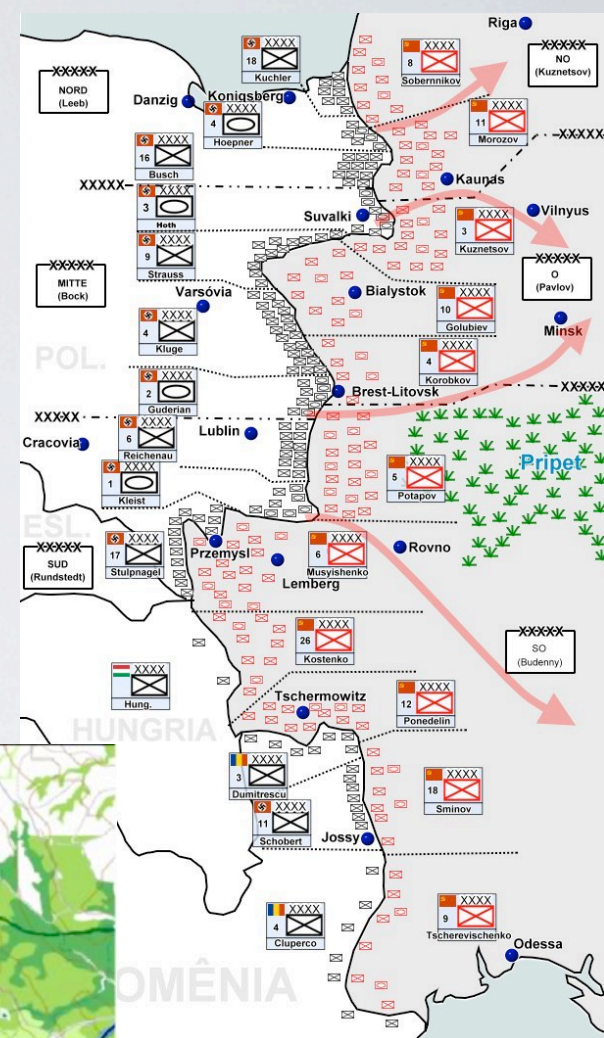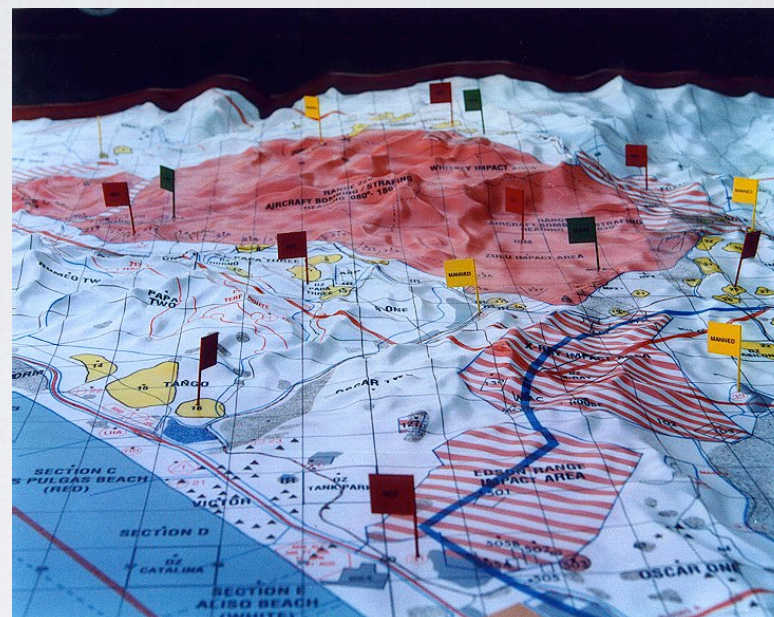
'The most interesting and challenging [C$^2$] endeavors are those that involve a *collection* of military and civilian sovereign entities with *overlapping* interests that can best be met by sharing information and collaboration that *cuts across the boundaries* of the individual entities.'

*Understanding Command and Control*, Alberts & Hayes, 2006.

# TOPOLOGY IN C²

- The Twin Curses of a C² Theory:

  *Dimensionality* and *Complexity* (Levis&Athans,1987)

- Dimensionality: **4 key domains** (Alberts,2009)

- Complexity: **their interrelationships**:

  - Conceptual Model: ≈Hundreds of variables   ≈**Thousands of relationships** !!!
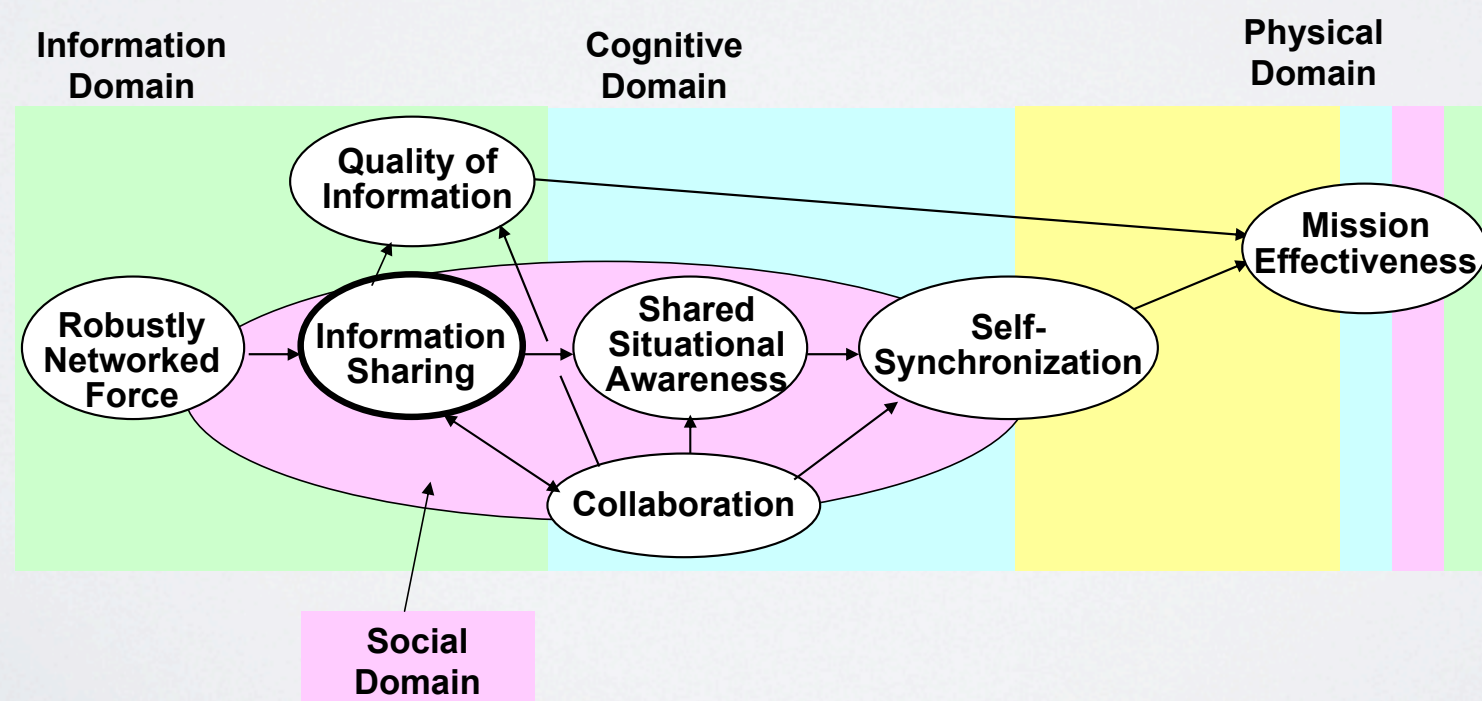
# TOPOLOGY IN C$^2$

- 'Limitations present in C$^2$ Analysis:

  - **there are no proper investigation tools;**

  - **there is no *theory*; and**

  - **the treatment of *structure* has been neglected.**'

  *Mathematics of Command and Control*, Dockery, 1984

# DEFINITIONS

- **Definition**: A *Topological Space* is a set **X** together with a collection **O** of subsets of **X**, called *open sets*, such that

    - the union of any collection of set in **O** is in **O**;

    - the intersection of any finite collection of sets in **O** is in **O**;

    - Both ∅ and **X** are in **O**.

    The collection **O** is called a *Topology*.

# DEFINITIONS

- **Definition**: An *Abstract Simplicial Complex* $\Delta$ on a finite set **X** is a family of subsets closed under deletion of elements. We refer to the singleton sets **x** in $\Delta$ as **0-simplices** or **vertices**. It is not required that $x \in \Delta$ for all $x \in X$.



0- Simplex $[v_0]$

1- Simplex $[v_0, v_1]$

2- Simplex $[v_0, v_1, v_2]$

3- Simplex $[v_0, v_1, v_2, v_3]$

Extracted from: GHRIST, R.; MUHAMMAD, A. *Coverage and hole-detection in sensor networks via homology*. In: IEEE PRESS. Proceedings of the 4th Intl.Symp. on Information Processing in Sensor Networks, 2005. p. 34.
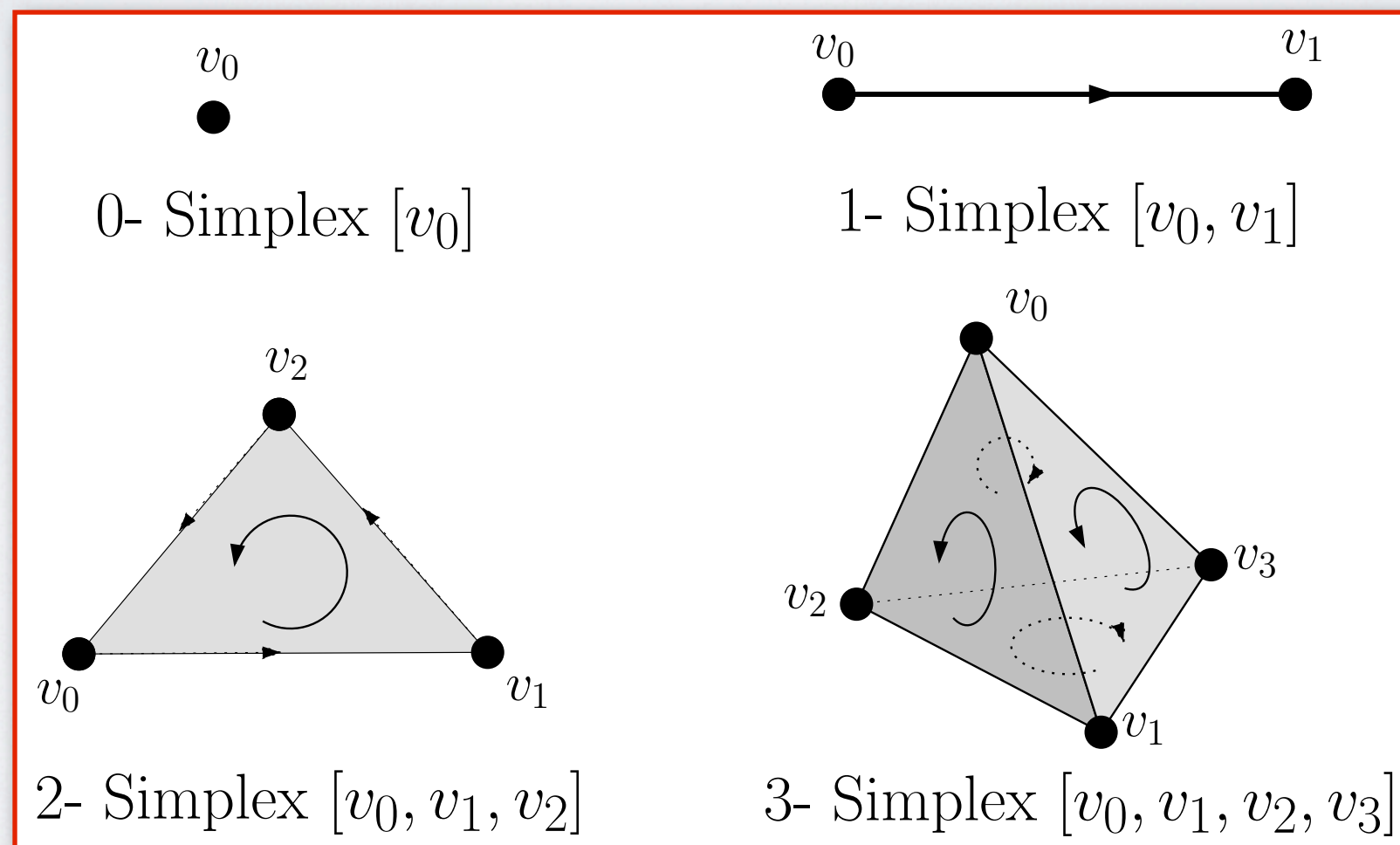
- **Basic elements**:

  - Roles (R)

  - Relationships (Rel)

  - Tasks (T)

$$O = (O_{struct}, O_{func}, O_{assign})$$
$$O_{struct} = (R, Rel)$$
$$O_{func} = (T, P(T))$$
$$O_{assign} = \{(r_i, \{t_i\}) | r_i \in R, t_i \in T\}$$

$$O_{behaviorstate} = (r_{i,k}, \{t_{i,k}\})$$

$$O_{behaviorstate,k} \longrightarrow O_{behaviorstate,k+1}$$

- A tiny example - Air Defense:

- 4 Roles

  - **CC** - Central Command Post

  - **LC** - Local Command Post

  - **F** - Fighter Aircraft

  - **AAA** - Anti-Aircraft Artillery

- 5 Tasks

  - **DI** - Detect and Identify

  - **OE** - Order to Engage

  - **IN** - Intercept

  - **EA** - Engage Artillery (AAA)

  - **RR** - Report Results

- Two steps process:

  - Step 1: **To build an** *interaction poset* **(partially ordered set)** *P* **from the task dependency graph**

$O_{struct}$ : ({CC,LC,F,AAA},{CC < LC,LC < F,CC < AAA})

$O_{func}$ : ({DI,OE,IN,EA,RR},{DI < OE,OE < IN,OE < EA,IN < RR,EA < RR})

$O_{assign}$ : {(CC, {DI, OE}), (LC, ∅), (F, {IN, RR}), (AAA, {EA, RR})}
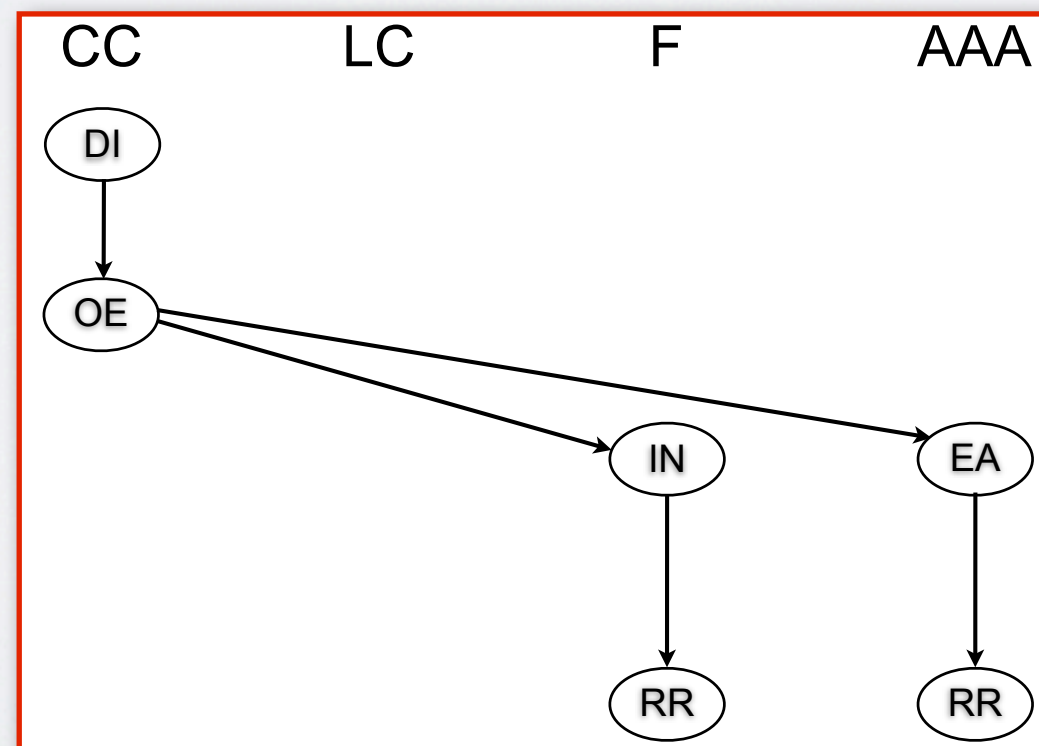
- **Definition**: An *Order Complex $\Delta_0(P)$* is a simplicial complex whose vertex set contains all elements of the interaction poset *P*. A subset of *P* is a simplex of $\Delta_0(P)$ if and only if its elements form a chain in *P*, i.e., they can be arranged to form a totally ordered subset of *P*.

  - Step 2: **To build the *order complex $\Delta_0(P)$* from the *interaction poset P***
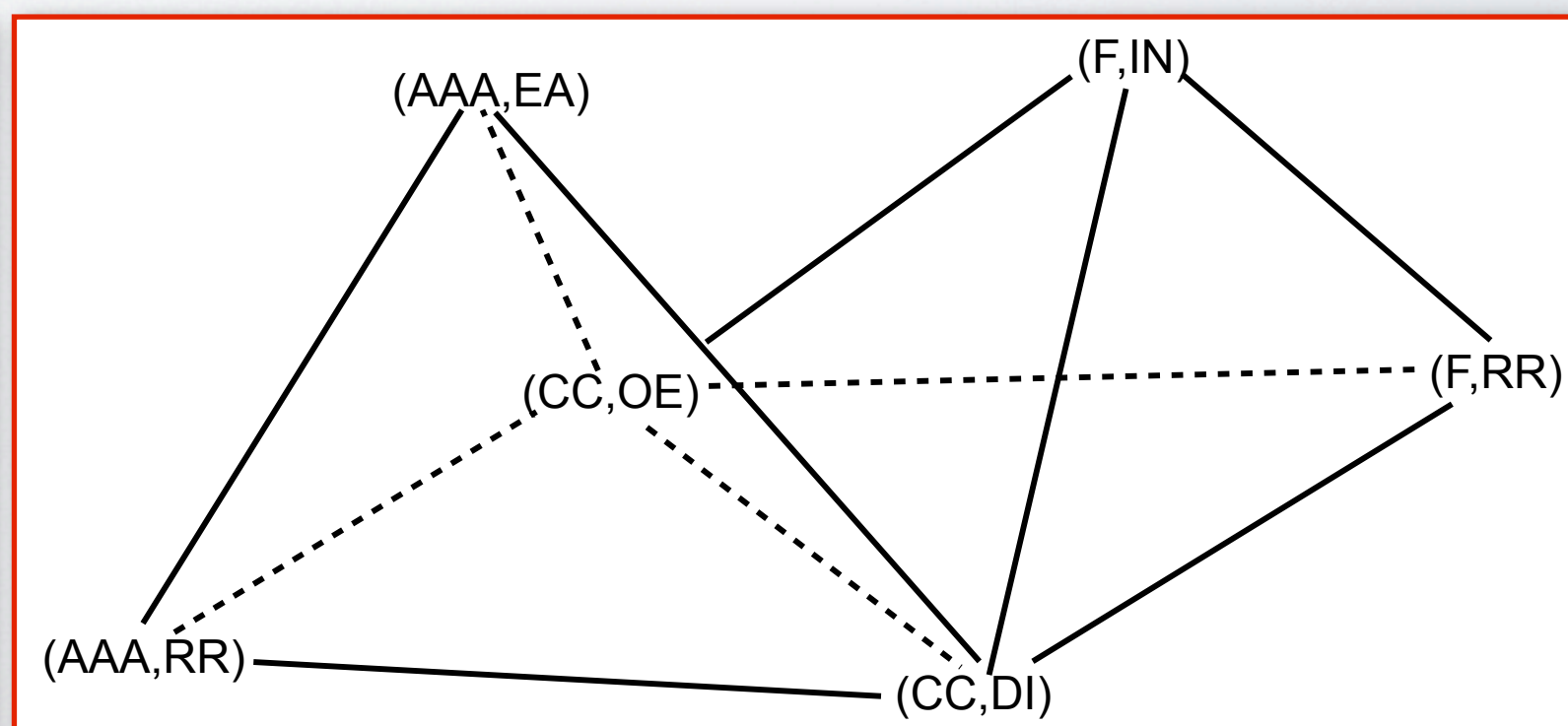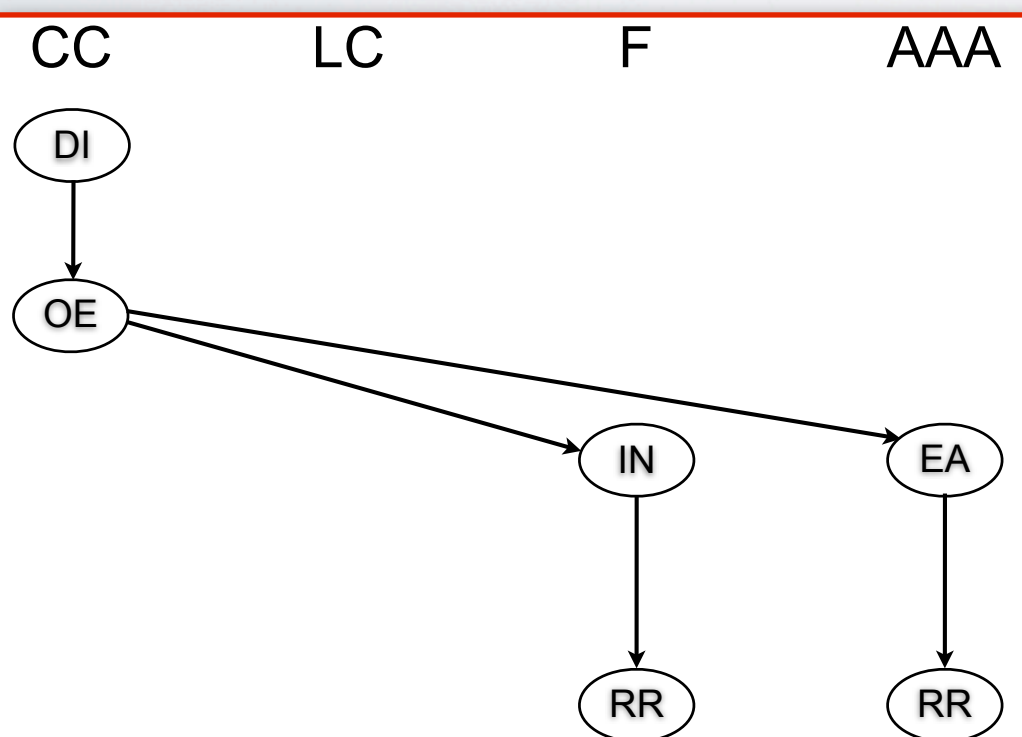
$$\Delta_0(P) = \{\{(CC,DI),(CC,OE),(F,IN),(F,RR)\}, \{(CC, DI), (CC, OE), (AAA, EA), (AAA, RR)\}\}$$

- Baseline organization:

$$O_{struct} : (\{CC,LC,F,AAA\},\{CC < LC,LC < F,CC < AAA\})$$
$$O_{func} : (\{DI,OE,IN,EA,RR\},\{DI < OE,OE < IN,OE < EA,IN < RR,EA < RR\})$$
$$O_{assign} : \{(CC, \{DI, OE\}), (LC, \varnothing), (F, \{IN, RR\}), (AAA, \{EA, RR\})\}$$

$$\Delta_o(P) = \{\{(CC,DI),(CC,OE),(F,IN),(F,RR)\},$$
$$\{(CC, DI), (CC, OE), (AAA, EA), (AAA, RR)\}\}$$
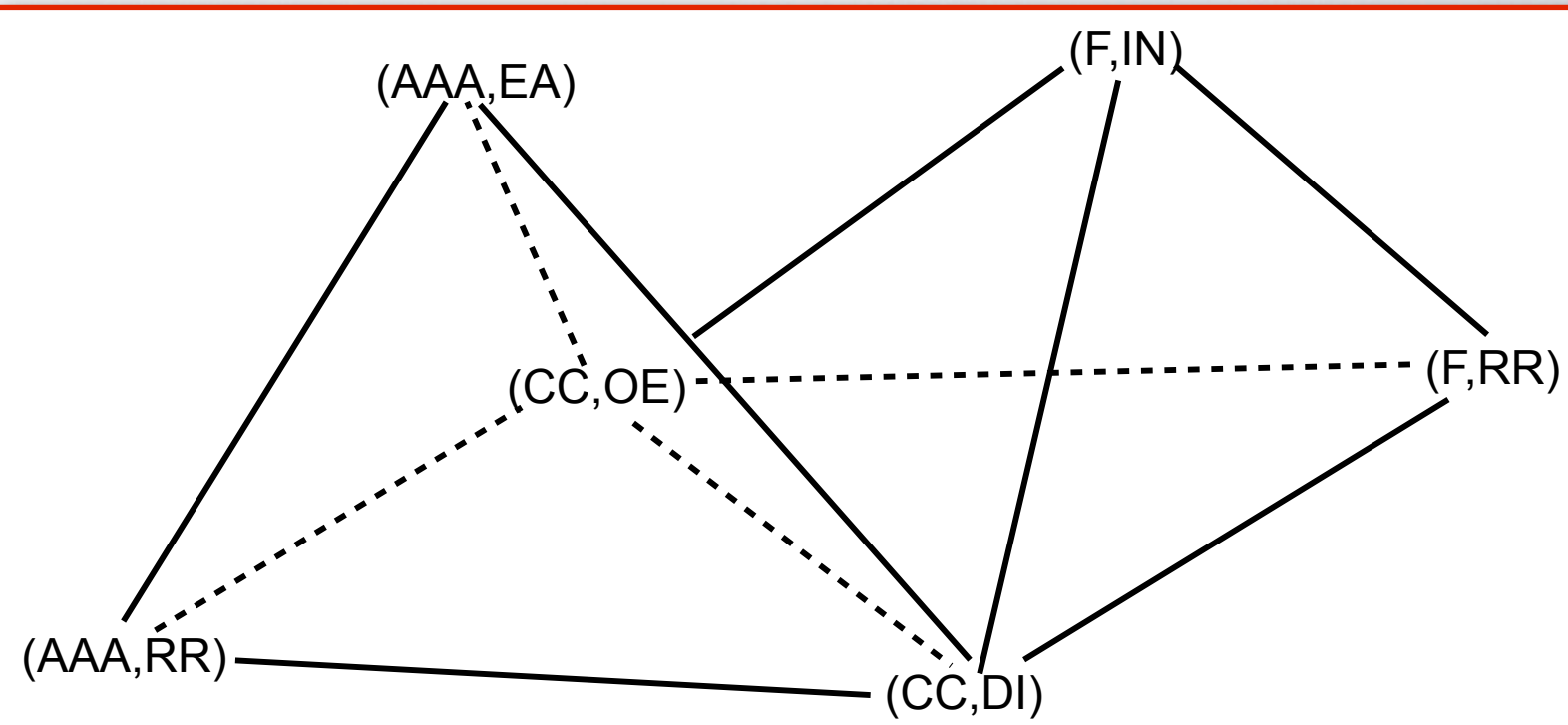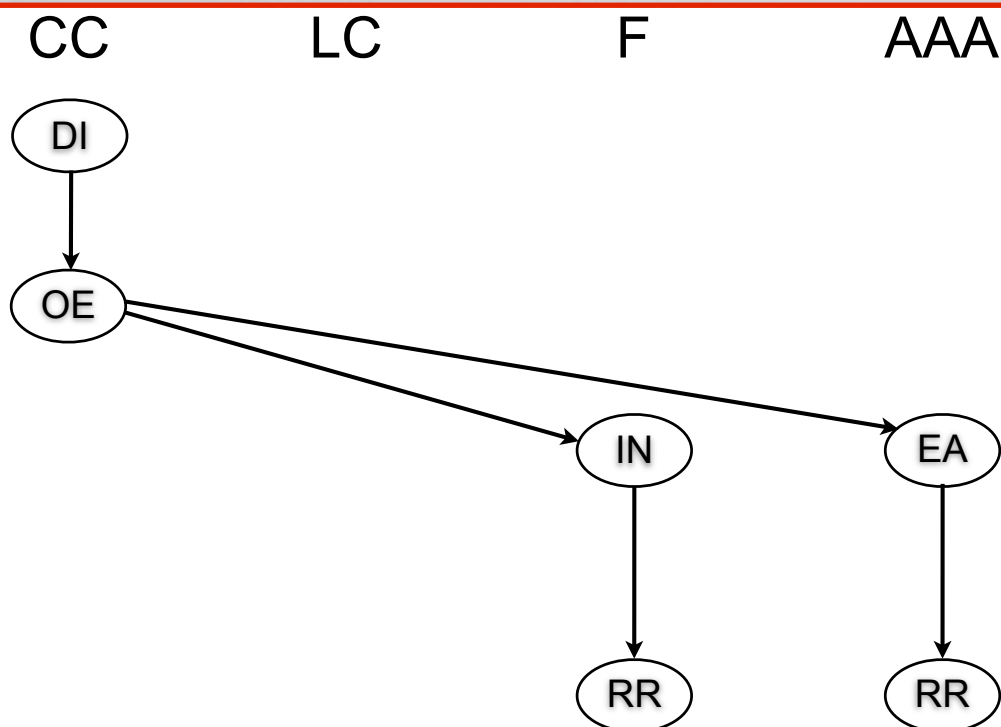
- Change 1: **CC** delegates to **LC** responsibility to order fighters.

$$O_{struct} : (\{CC,LC,F,AAA\},\{CC < LC,LC < F,CC < AAA\})$$
$$O_{func} : (\{DI,OE,IN,EA,RR\},\{DI < OE,OE < IN,OE < EA,IN < RR,EA < RR\})$$
$$O_{assign} : \{(CC, \{DI, OE\}), (LC, \{OE\}), (F, \{IN, RR\}), (AAA, \{EA, RR\})\}$$
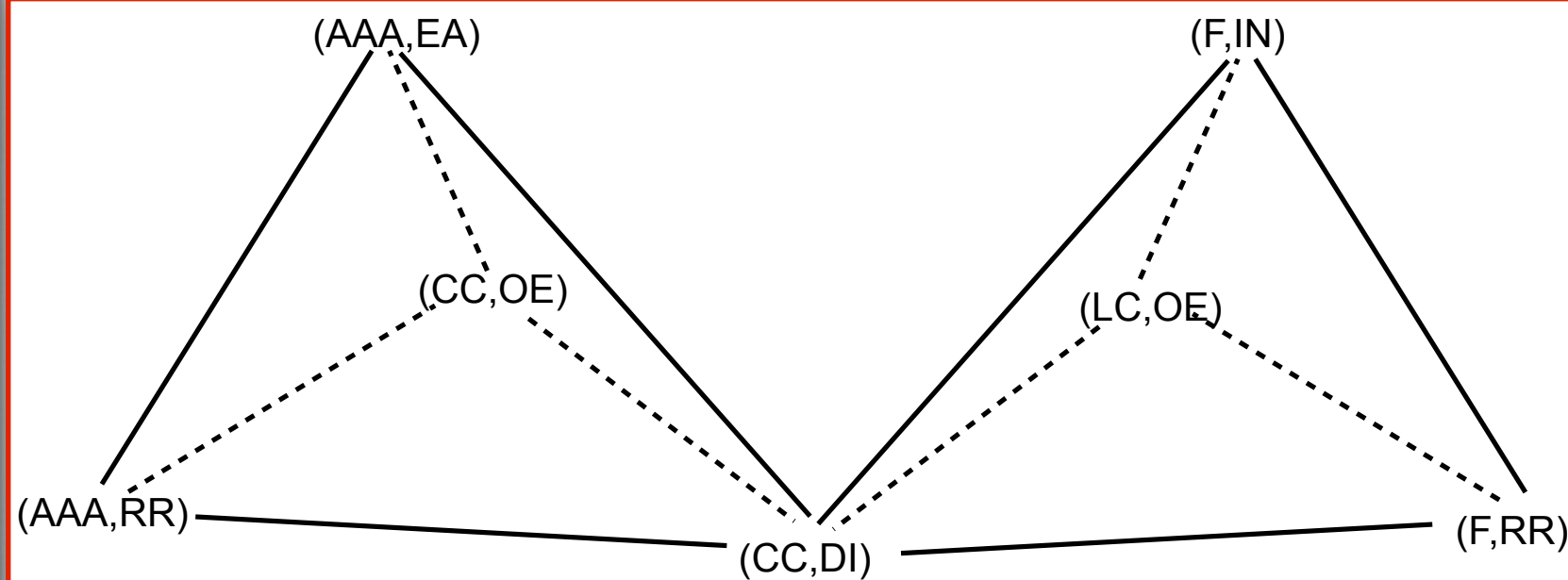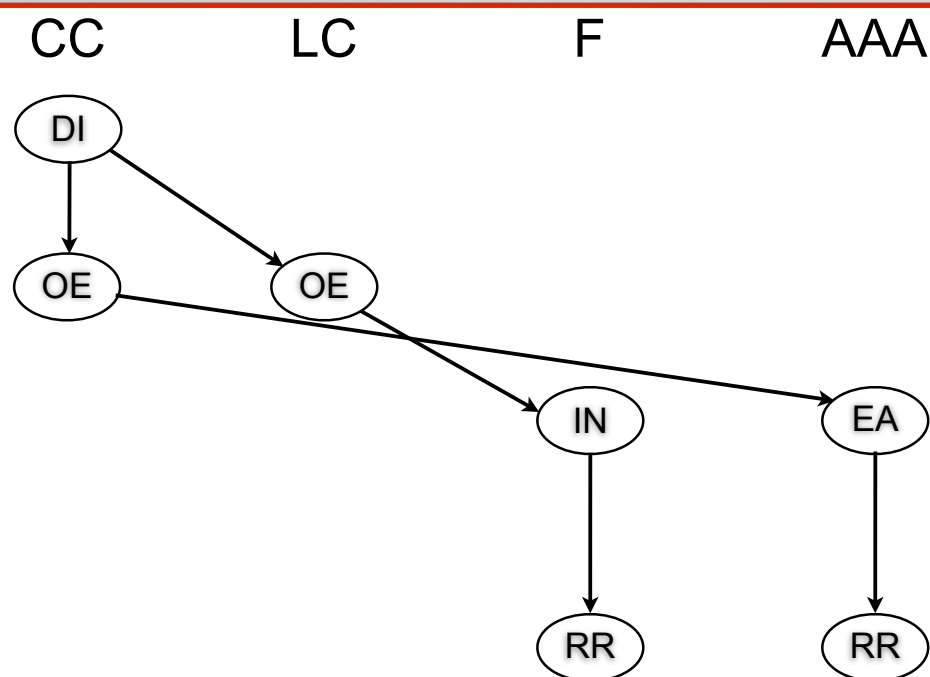
$$\Delta_o(P) = \{\{(CC,DI),(LC,OE),(F,IN),(F,RR)\},$$
$$\{(CC, DI), (CC, OE), (AAA, EA), (AAA, RR)\}\}$$

- Change 2: **LC** now also commands **AAA**

$$O_{struct} : (\{CC,LC,F,AAA\},\{CC < LC, LC < F, CC < AAA\})$$
$$O_{func} : (\{DI,OE,IN,EA,RR\},\{DI < OE, OE < IN, OE < EA, IN < RR, EA < RR\})$$
$$O_{assign} : \{(CC, \{DI\}), (LC, \{OE\}), (F, \{IN, RR\}), (AAA, \{EA, RR\})\}$$
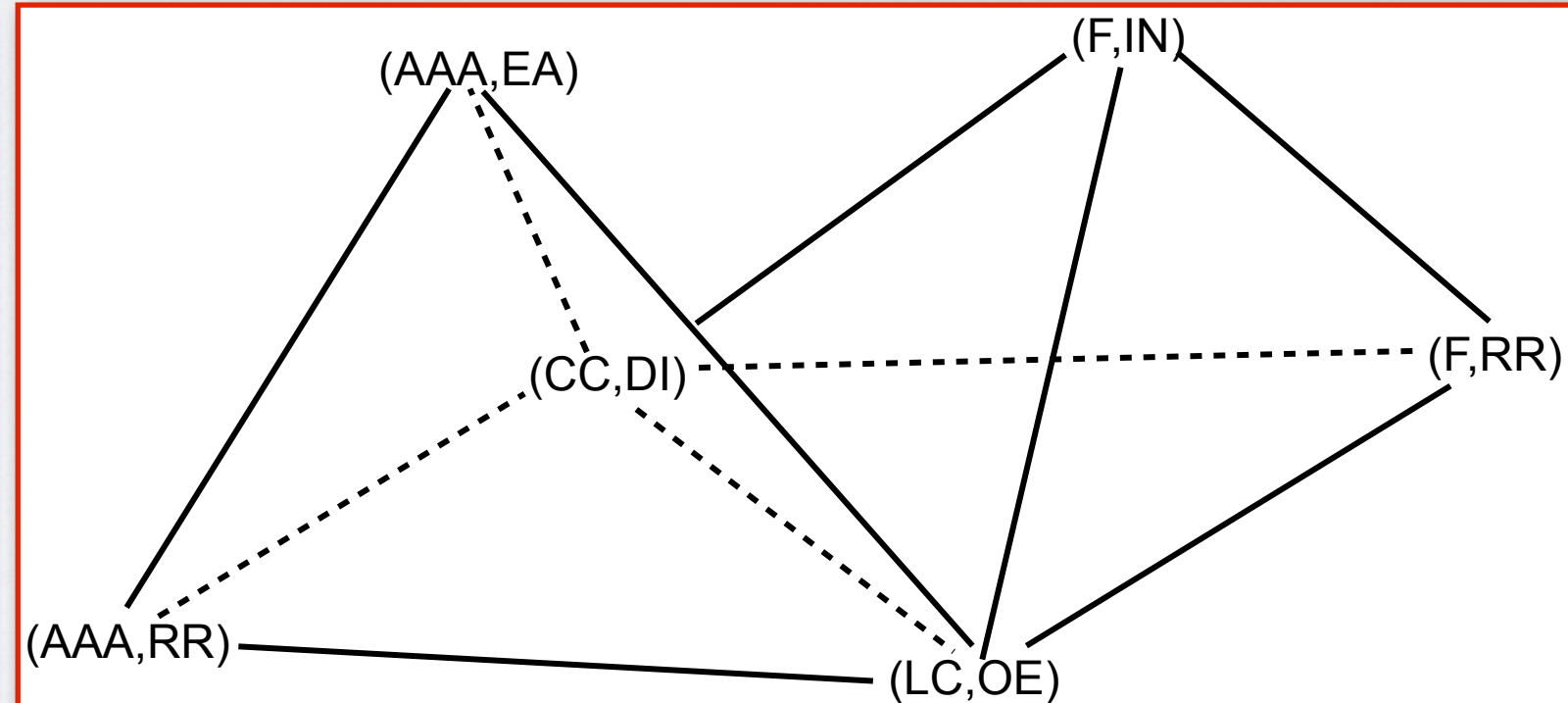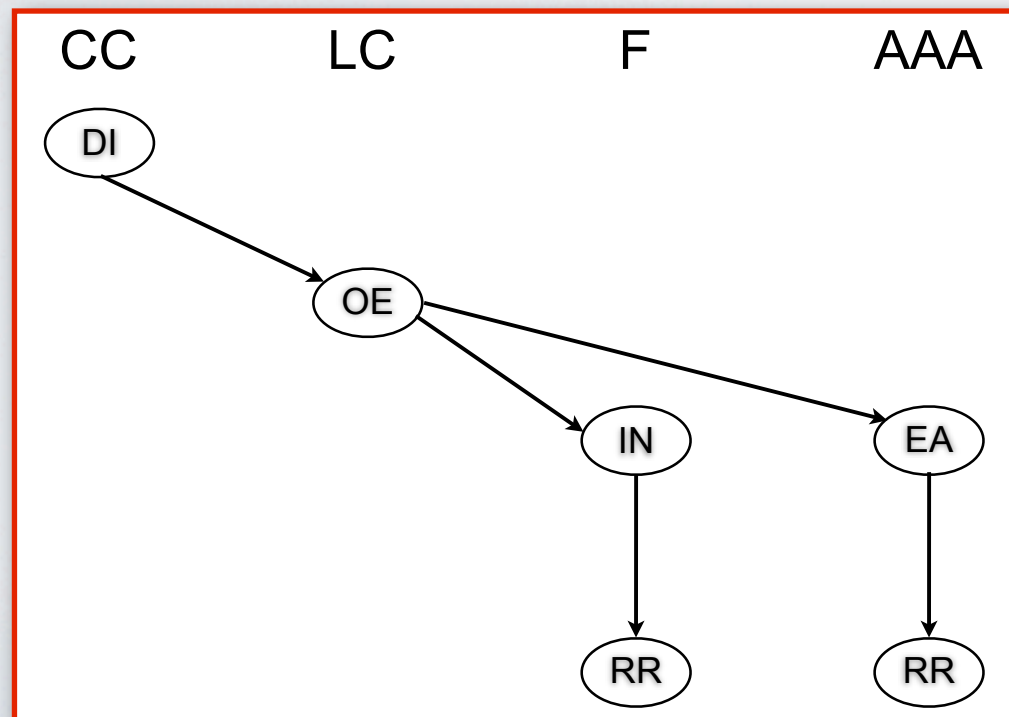
$$\Delta_o(P) = \{\{(CC,DI),(LC,OE),(F,IN),(F,RR)\},$$
$$\{(CC, DI), (LC, OE), (AAA, EA), (AAA, RR)\}\}$$

- Change 3: **AAA** goes autonomous...

$$Ostruct : (\{CC,LC,F,AAA\},\{CC < LC,LC < F,CC < AAA\})$$
$$Ofunc : (\{DI,OE,IN,EA,RR\},\{DI < OE,OE < IN,OE < EA,IN < RR,EA < RR\})$$
$$Oassign : \{(CC, \{DI\}), (LC, \{OE\}), (F, \{IN, RR\}), (AAA, \{DI, OE, EA, RR\})\}$$

$$\Delta o(P) = \{\{(CC,DI),(LC,OE),(F,IN),(F,RR)\},$$
$$\{(AAA,DI), (AAA,OE), (AAA, EA), (AAA, RR)\}\}$$
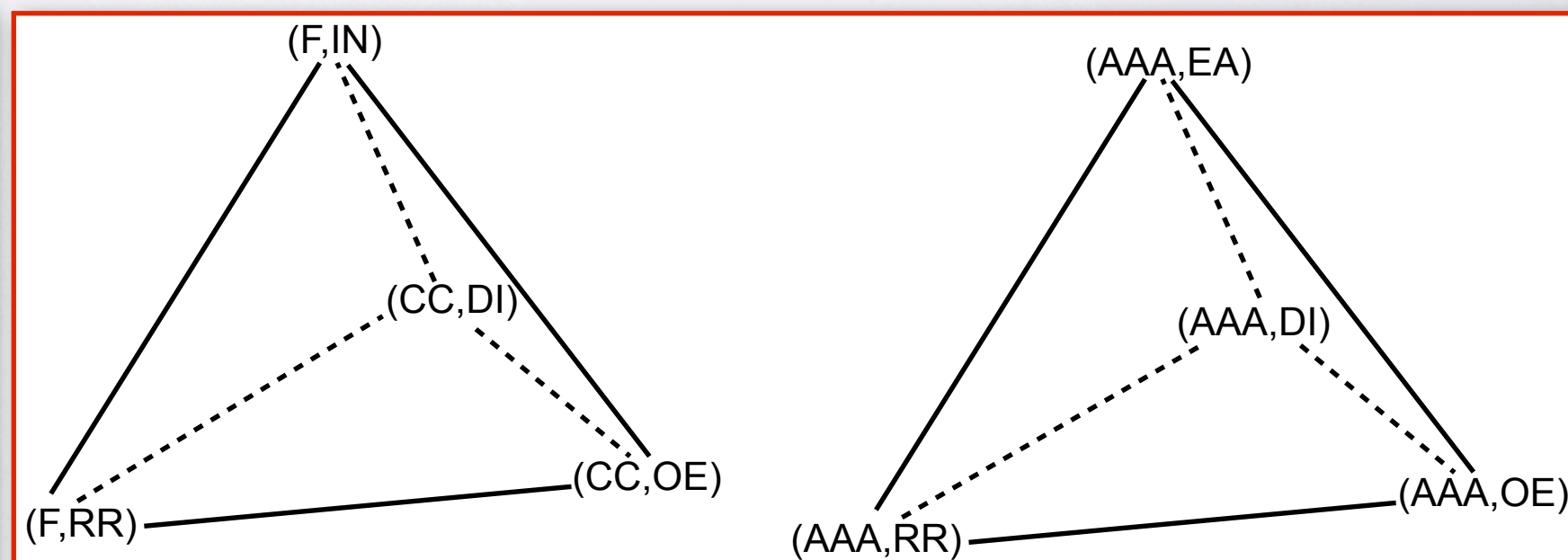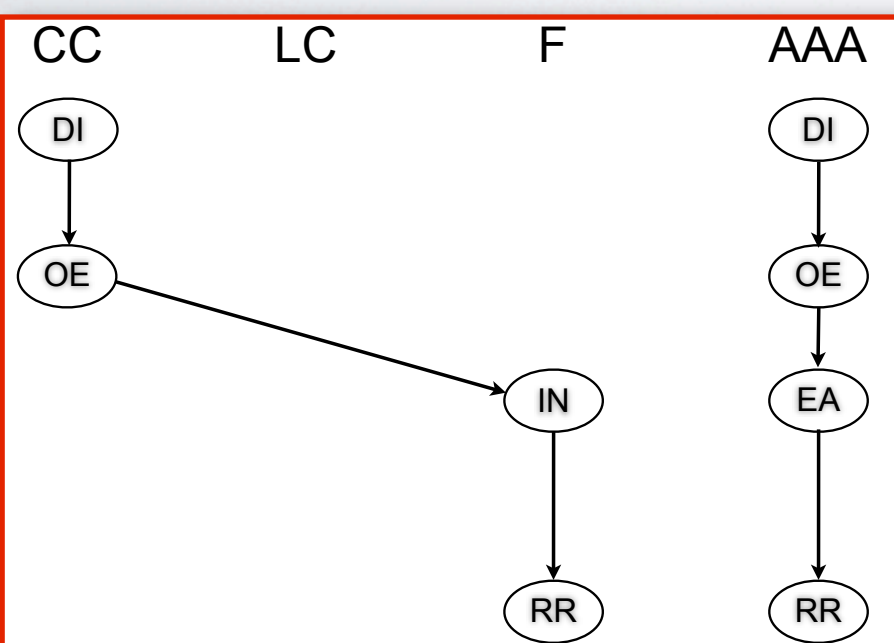
- Dynamics:

  - Baseline:

  - Change 1:

  - Change 2:

  - Change 3:
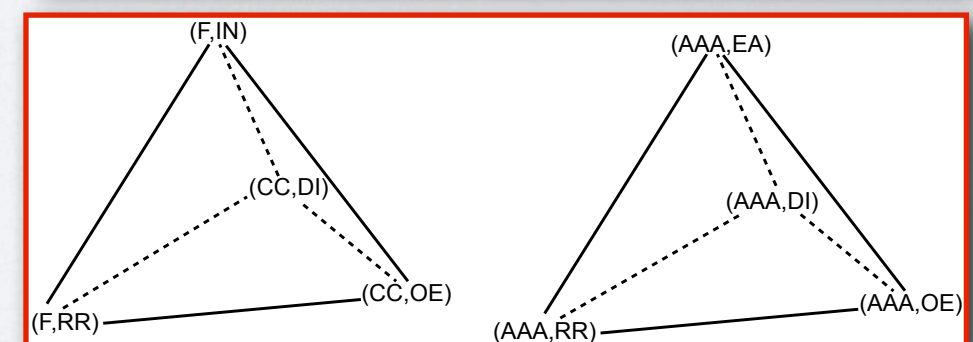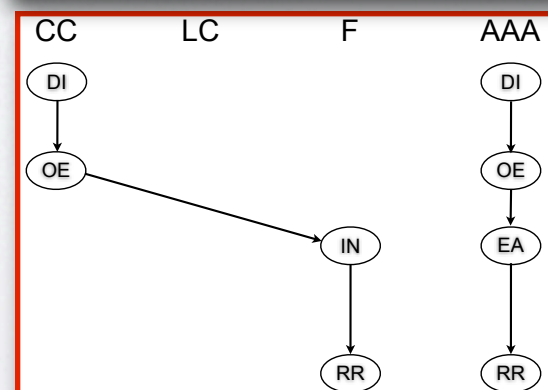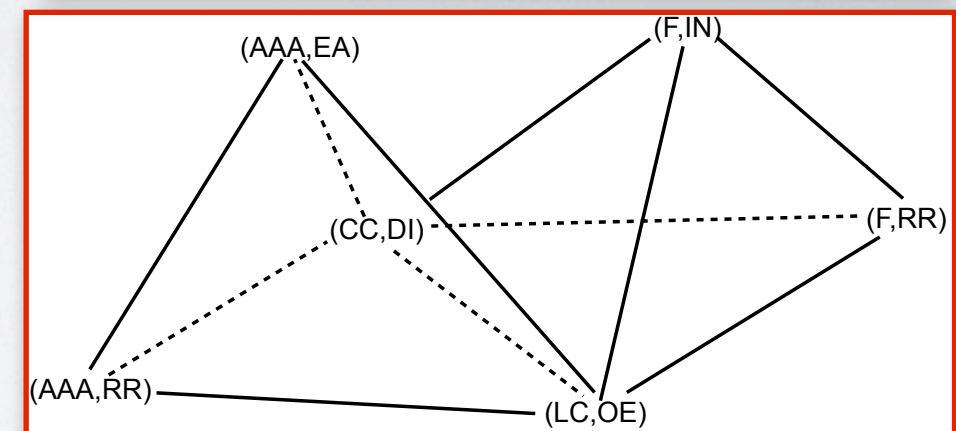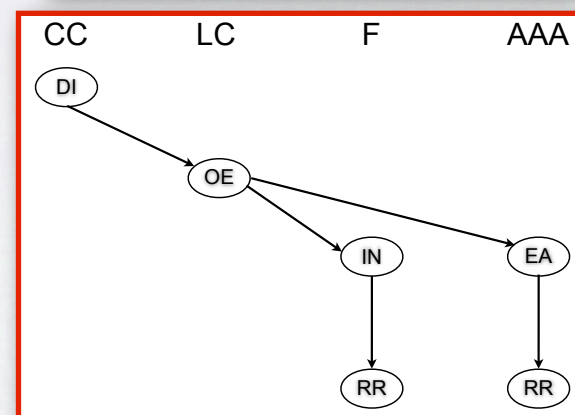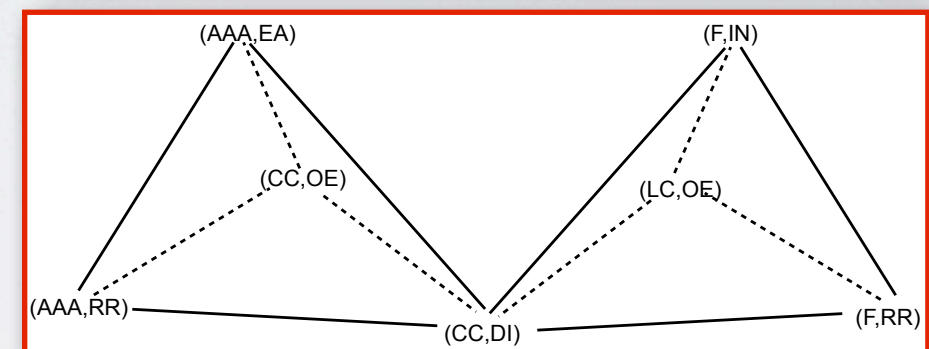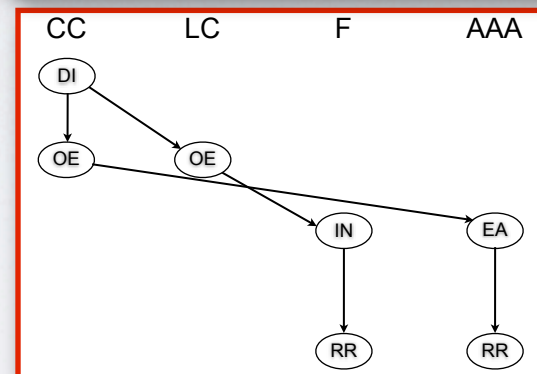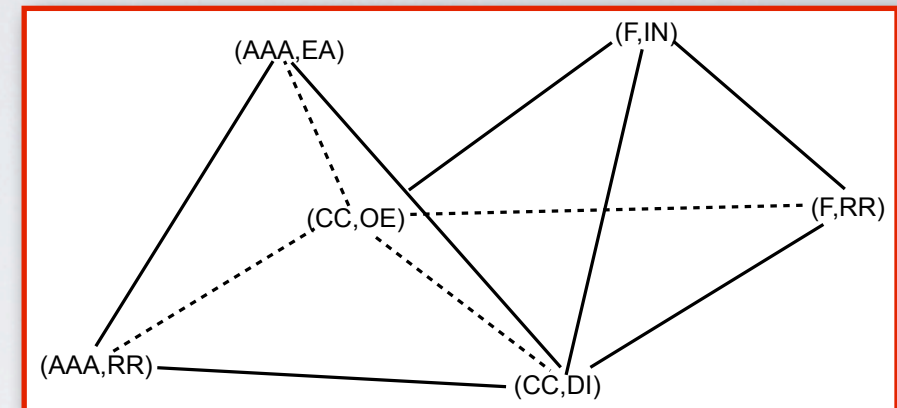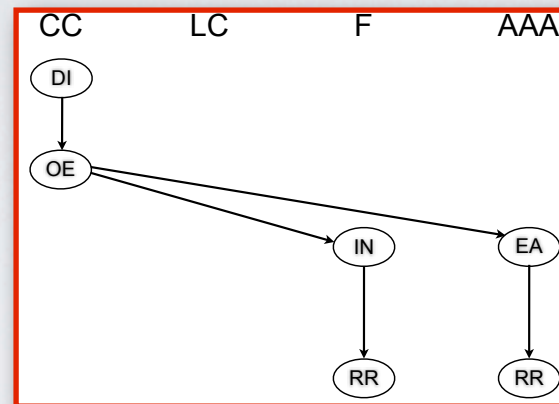
# CONCLUSIONS

- A topological model of C2 organizations, based on simplicial complexes, that captures tasks' dependencies and distribution, was presented;

- Simplicial complexes have the main advantage to capture higher dimensional relationships, where graphs are only unidimensional;

- By changing task dependencies or task allocation the connectivity of the resulting simplicial complex changes;

- It means, the *combinatorics* related to the behavior of these organizations is changed!

# CONCLUSIONS

- Next steps:

    - Information flow (*concurrency*)

    - Geospatial and temporal topological relationships

    - Power relationships

# CONCLUSIONS

- *Main tenets of this approach:*

    - Any $C^2$ organizational model will have to deal with the *combinatorics* of multidimensional parameters (structure, functions, environment, cognition, capabilities and resources);

    - Maybe the relationships between these parameters cannot be captured in a graph style, one dimension only;

    - *Simplicial complexes* are an adequate mathematical construct to capture these higher dimensional relationships.

'We have therefore searched for a **theory**. More fundamentally, we have searched for a starting point for a theory. In the end we focused on **structural aspects** of $C^2$. In fact structure is but one of three aspects of the problem which we have identified. The other two are **data/information** and **transactions**. The complete characterization is therefore *transactions within a structure involving the flow of data/information through that structure*.'

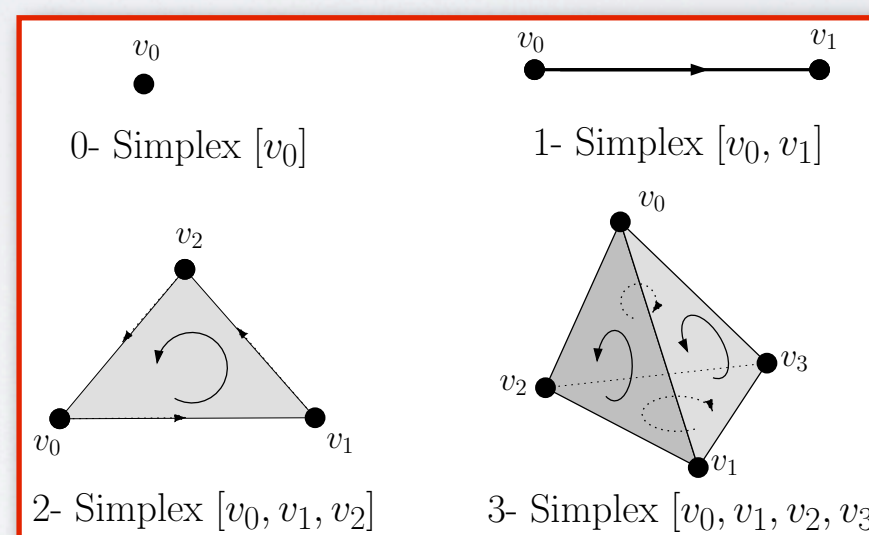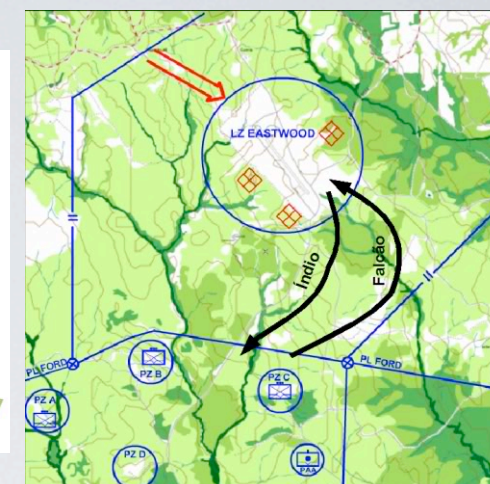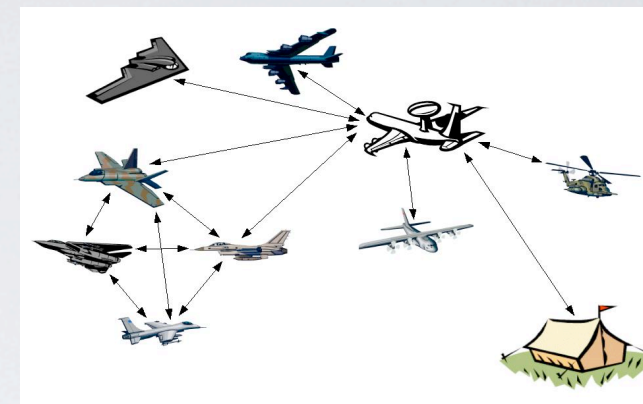*Mathematics of Command and Control*, Dockery, 1984

# SUMMARY

- Topology in the military

- Topology in C$^2$

- Definitions

- The *Model*
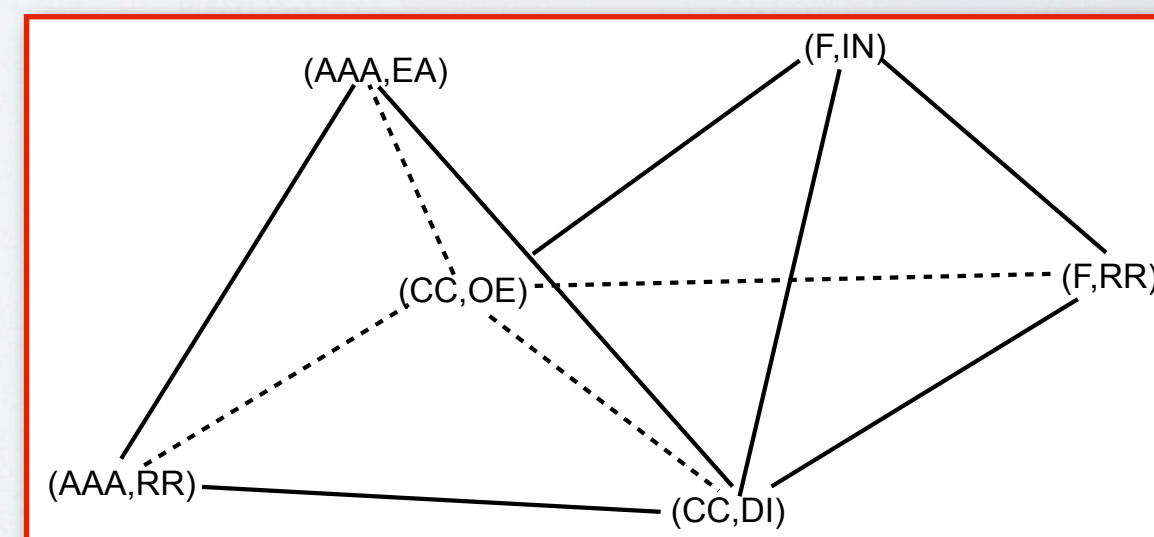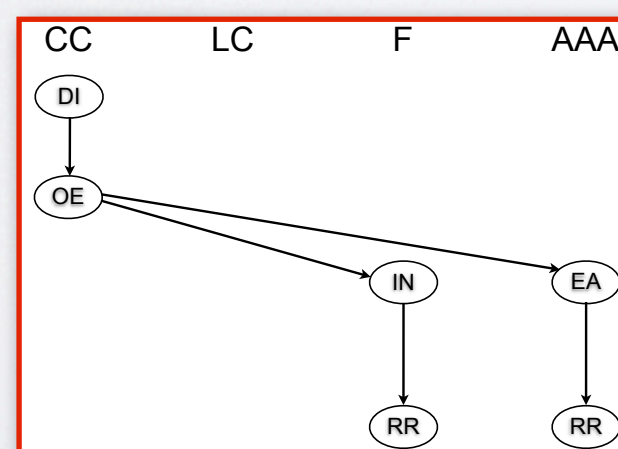
# OBJECTIVES

- *To present a topological model of $C^2$ organizations*

- *To motivate the $C^2$ research community for the application of Topology to $C^2$ theory building*

# A TOPOLOGICAL MODEL OF C² ORGANIZATIONS

*Lt.Col. André Luiz Pimentel Uruguay, PhD Student*
Instituto de Estudos Avançados - Brazilian Air Force
**auruguay@gmail.com**
*Prof. Carlos Henrique Costa Ribeiro, PhD*
Instituto Tecnológico de Aeronáutica - Brazilian Air Force
**carlos@ita.br**